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CITY AND GUILDS OF LONDON INSTITUTE EXAMINATIONS, 1966

QUESTIONS AND ANSWERS

Answers are occasionally omitted or reference is made to earlier Supplements in which questions of substantially the same form, together with the answers, have been published. Some answers contain more detail than would be expected from candidates under examination conditions.

TELECOMMUNICATION PRINCIPLES A, 1966 (*continued*)

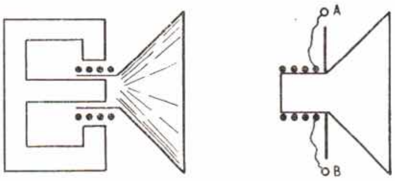
Q. 9. *Describe the principle of operation of the moving-coil loudspeaker. What factors determine*

(a) the sensitivity,

(b) the distortion produced by this type of speaker?

The moving coil of the loudspeaker consists of 100 *turns in the form of a cylinder* 5 *cm in diameter. This moves axially along a cylindrical air-gap across which is maintained a uniform radial magnetic field of intensity* 0·5 *Wb/m*^{2}. *Calculate the axial deflecting force on this coil per milliampere of current.*

A. 9. The principle of the moving-coil loudspeaker can be described as an application of the "motor principle". When a coil carries a current that cuts a magnetic field, the coil experiences a force proportional to the current.



A strong radial magnetic field is maintained across an annular air-gap by means of a permanent magnet in the form, usually, of a cylinder with poles at the ends. The poles are extended by (*i*) a soft iron disk with an axial extension to form the inner pole face, N, and (*ii*) a soft-iron disk with a large central hole to form the outer pole face, S. The lines of force are uniformly distributed around the air-gap, all being radial.

A coil of wire on a cardboard (or moulded) cylindrical former moves freely in the air-gap, the turns of the coil being circumferential in the gap. Flexible leads connect the coil to fixed terminals AB in the sketch.

The coil is fixed concentrically to a conical diaphragm so that any movement of the coil is imparted to the cone. The cone is supported freely by a ring of loose linen material around its outer circumference on a "baffle" board having a hole of suitable size to admit the cone. A more constricting support is by a "spider" of thin card at the neck of the cone permitting considerable axial movement but no sideways motion.

When an alternating current passes through the coil, a force is generated on it oscillating at the frequency of the current. The magnitude of the force is proportional to the magnitude of the current. The cone is therefore forced to vibrate in sympathy and its large area sets up sound pressure waves in the air, transferring the energy of motion from the coil to the air.

(*a*) The sensitivity of the moving-coil speaker per unit current depends on the total magnetic flux and the length of the conductor on the coil. The efficiency of the cone in "coupling" to the air also affects the volume of sound obtained.

(*b*) The distortion, i.e. alteration of frequencies and their relative levels from those in the electrical input, chiefly depends on the uniformity of the flux whatever the linear movement of the coil. Since the permanent magnet is expensive, it is usual to keep its field to the minimum acceptable by making the moving coil longer than the axial length of the air-gap. If the turns of the coil are uniformly spaced, a constant number of turns are then always in the magnetic field whenever the coil is moving to-and-fro. This keeps down distortion at full load current.

It is necessary to avoid resonances in the moving coil and cone. The cone must move solidly like a piston, against the air. It is especially important that it should do this at low frequencies (i.e. long wavelengths of sound in air) where it is less efficient in launching sound waves.

Axial deflecting force on the coil per mA = Intensity of field × length of wire × current

$$= 0{\cdot}5 \times \pi (5 \times 10^{-2}) \times 100 \times 10^{-3}$$

$$= \underline{2{\cdot}5\,\pi \times 10^{-3}} \text{ newtons.}$$

Q. 10. *Explain the principle of the simple potentiometer when used as a means of comparing e.m.f. Why is it important that the slide wire should be uniform throughout its length?*

A small direct current flows in a 100-*ohm resistor. How could the potentiometer be used to measure this current without breaking the circuit?*

If the standard cell of 1·018 *volts gives a balance against half the length of the slide wire, what proportion of the length would correspond to the measurement of* 15·3 *mA in the* 100-*ohm resistor?*

A. 10. A potentiometer, which is a device for measuring potential difference, provides an accurate method of comparing two voltages.

It uses a null method of measurement, which means that no current is drawn from the circuit under test at the instant of recording the voltage. The voltage measurement therefore leaves the circuit undisturbed, so that its voltage is not changed by the measurement.

(*b*) A long-distance, overseas, point-to-point radio-telephony service would use carrier frequencies in the high frequency range between 3–30 Mc/s, more especially between 4–25 Mc/s and employ single sideband or independent sideband transmissions. In either case the radio-frequency bandwidth occupied by each telephone channel would be 3 kc/s, such a bandwidth being adequate for commercial speech.

(*c*) To provide a short-distance broadcast service a carrier frequency in the v.h.f. band, normally between 80–100 Mc/s would be used. For a high quality broadcast service the bandwidth would be about 30 kc/s for amplitude-modulated signals and about 150 kc/s for frequency-modulated signals.

(*d*) A private mobile radio-telephone service would use carrier frequencies in the v.h.f. range, more especially between 80 and 160 Mc/s. Double sideband amplitude modulation or frequency modulation may be employed utilizing a channel bandwidth of 25 kc/s.

The velocity of propagation v is related to its frequency f and wavelength as follows,

$$v = f \times \lambda$$

When v is in metres/second; λ is in metres and f is in cycles/second.

Hence, $f = \frac{v}{\lambda}$ and substituting the values given, the assigned frequency of the transmitter $= \frac{3 \times 10^8}{37{\cdot}5}$ c/s = 8,000,00 c/s.

The working frequency is found to be 8,000.21 kc/s = 8,000,210 c/s. Hence the deviation is 8,000,210 − 8,000,000 c/s.

$$= \underline{210 \text{ c/s.}}$$

This deviation, expressed as a fraction of the assigned frequency,

$$= \frac{210}{8{,}000{,}000} = 26{\cdot}25 \times 10^{-6}.$$

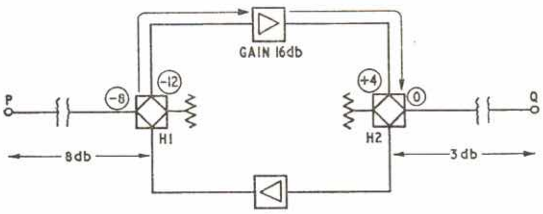
This is within the permitted tolerance of ± 30 parts in 10^6.

Q. 6. *By reference to a block schematic diagram, outline the principle of operation of a repeatered audio junction circuit on* (a) *a 2-wire basis,* (b) *a 4-wire basis.*

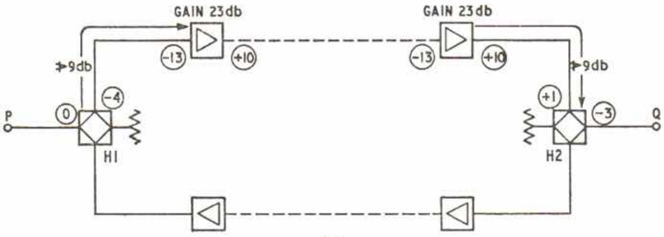
Briefly explain the purpose of the hybrid transformers.

Quote typical gains and losses for the various parts of the circuit.

A. 6. Sketches (*a*) and (*b*) show the required layout for 2-wire and 4-wire repeatered audio junction circuits, respectively. In both types of circuit, two-way signals pass along the lines between point P and hybrid transformer H1, and between hybrid transformer H2 and point Q. The hybrid transformers are arranged to transform the circuits from 2-wire to 4-wire. Thus, in sketches (*a*) and (*b*), signals from P to Q are assumed to pass along the top part of the loops and those from Q to P to pass along the bottom part of the loops. In this way the 2-way signals are split into unidirectional signals to meet the requirements of the amplifiers. The 2-wire circuit derives its name because the circuit between P and Q is principally 2-wire and a 4-wire circuit is only used at the points of amplification. The 4-wire circuit is so-called because, apart from the short length of 2-wire circuits at each end, it is principally a 4-wire circuit throughout its length.

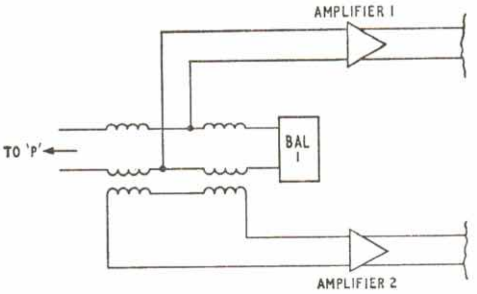


(*a*)



(*b*)

Sketch (*c*) shows the hybrid transformer in more detail and the manner in which it effects the change from 2-wire to 4-wire working. If the balancing network BAL 1 has the same impedance as the 2-wire line connected to the hybrid transformer then it may be shown that the hybrid transformer is balanced and the output from amplifier, AMP 2 will not develop a voltage at the input of amplifier AMP 1.



(*c*)

Thus, the second purpose of the hybrid transformer is to arrange that there is no feedback of signal around the loop formed by the hybrid transformer and the amplifiers. In practice it is not possible to arrange a perfect balance and other steps may need to be taken to reduce the effects of any unbalance.

Typical figures for the losses and gains in the various parts of the 2-wire and 4-wire circuits are given in sketches (*a*) and (*b*). The ringed figures show typical signal levels at the various points, expressed in decibels relative to one milliwatt.

Q. 7. *Explain, by reference to a frequency-response curve, how a tuned circuit may be used to provide selectivity in a radio-frequency amplifier.*

The measured response of a parallel-tuned circuit is as shown in Table II.

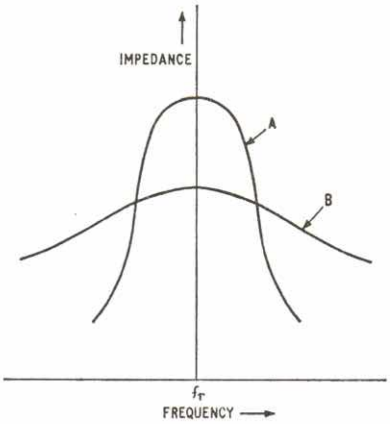
Table II

Frequency in kc/s	245	246	248	249	250	251	252	254	255
Voltage	35	55	83·5	96	100	96	83·5	55	35

Plot the response curve and use it to determine the bandwidth of the tuned circuit at the half-power (—3db) points.

Briefly comment on the suitability of such a circuit for use in a medium-wave broadcast receiver.

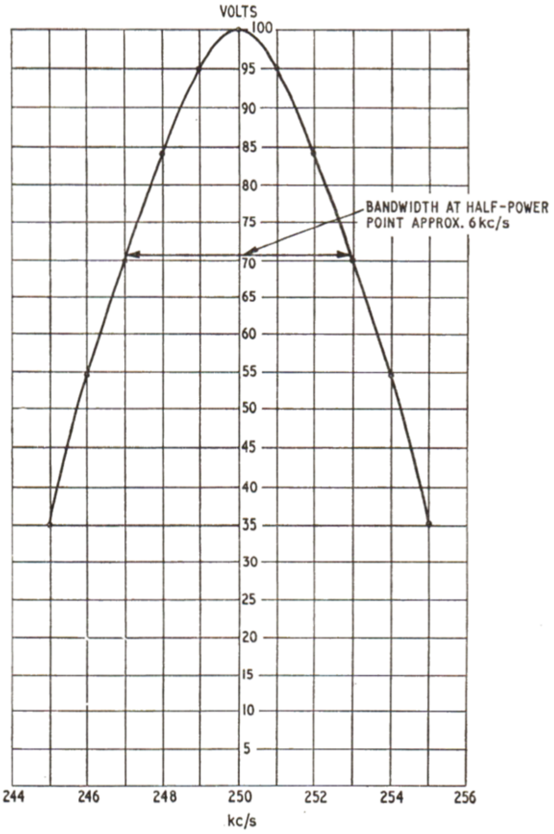
A. 7. Two examples of the variation of impedance with frequency for a parallel-tuned *LC* circuit are shown in sketch (*a*). These curves show that the impedance of a parallel-tuned *LC* circuit is a maximum at the frequency of resonance, f_r, and decreases in value on either side of f_r. This characteristic of a parallel-tuned *LC* circuit may be used in a receiver by connecting such a circuit as the anode load in the radio-frequency amplifier stage of the receiver. Since the voltage amplification-factor of a radio-frequency amplifier is almost directly proportional to



(*a*)

the value of its anode-load impedance, a desired signal at the resonant frequency of the anode load will be amplified more than undesired signals received at frequencies off resonance. This property of the tuned circuit, known as selectivity, is dependent upon the slope of the curve on either side of the resonance frequency. Thus, referring to sketch (*a*), curve A has a greater selectivity than curve B.

The response curve is given in sketch (*b*).



(*b*)

The half-power (−3 db) point may be found by using the relationship that

$$N = 10 \log_{10} \frac{P_1}{P_2} \text{ decibels} = 20 \log \frac{V_1}{V_2} \text{ decibels.}$$

For the half-power (−3 db) point,

$$-3 = 20 \log \frac{V_1}{V_2},$$

$$\log \frac{V_1}{V_2} = \frac{-3}{20} = -0{\cdot}15\,.$$

$$\text{or, } \log \frac{V_2}{V_1} = 0.15\,.$$

$$\therefore \frac{V_2}{V_1} = 1{\cdot}413\,.$$

Since $V_2 = 100$ volts then $V_1 = \frac{100}{1{\cdot}413} = 70{\cdot}7$ volt.

Referring to the response curve, the bandwidth at a voltage of 70·7 volts is given by AB which represents a bandwidth of 6 kc/s.

Alternatively, Since $P \propto V^2$, then $V \propto \sqrt{P}$.

Thus, if $\frac{P_1}{P_2} = \frac{1}{2}$, then $\frac{V_1}{V_2} = \frac{1}{\sqrt{2}} = 0{\cdot}7071$.

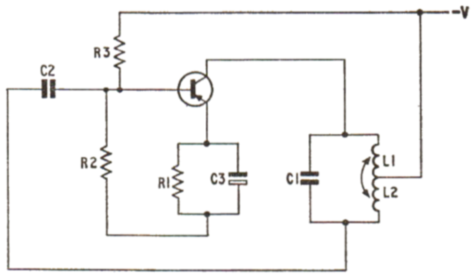
$\therefore V_1 = 100 \times 0{\cdot}7071 = 70{\cdot}7$ volts, as before.

In order to accommodate a reasonable number of radio broadcast transmissions in the long and medium wavebands it has been agreed internationally that the carrier frequency spacing shall be 9 kc/s. Hence, the highest audio frequency transmitted is 4·5 kc/s; this gives reasonable quality when music is broadcast. Since the response curve shown in sketch (*b*) only has a total bandwidth of 6 kc/s at the half-power points, such a circuit would give rather poor quality reproduction when used in a broadcast receiver, particularly for the reception of music.

***Q.* 8.** *Sketch the circuit and outline the operation of a simple mutual-inductance coupled transistor oscillator. Explain how such an oscillator may be modulated by an audio-frequency signal.*

Sketch the waveform of a radio-frequency wave amplitude modulated by a sinusoidal tone to a modulation depth of (a) 50 per cent, (b) 75 per cent. Axes should be clearly labelled to show the magnitudes involved.

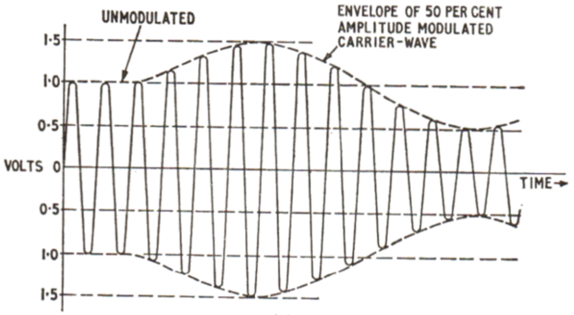
A. 8. Sketch (*a*) shows the circuit of a simple mutual-inductance coupled transistor oscillator of the Hartley type. The tuned circuit L1, L2, C1 is connected in the collector circuit of the transistor and the frequency of oscillation is very nearly given by $f = \frac{1}{2\sqrt{(L_1 + L_2)\,C_1}}$.



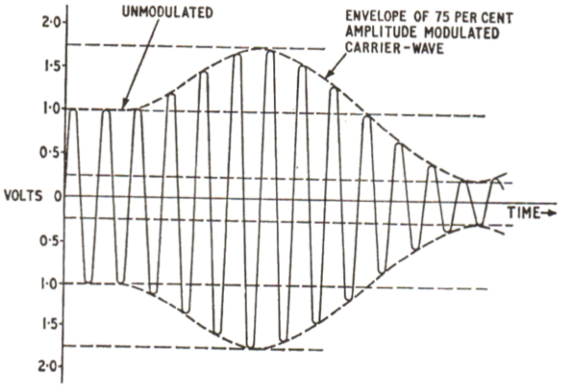
(*a*)

The circuit is, in effect, a common-emitter amplifier, with oscillations being maintained by inductive coupling between L1 and L2 and feedback, via C2, to the base circuit. D.C. stabilization of the operating point is provided by the potential divider circuit R2 and R3 and the emitter resistor R1. C1 is a decoupling capacitor.

The output of such an oscillator may be amplitude modulated by applying an audio-frequency signal to the base circuit by means of an audio-frequency transformer.



(*b*)



(*c*)

Sketches (*b*) and (*c*) show a radio-frequency wave amplitude-modulated by a sinusoidal tone to modulation depths of 50 per cent and 75 per cent, respectively.

Q. **9.** *Fig. 2 shows the circuit of a germanium-diode detector. Describe by reference to current and voltage waveforms, its use for demodulation of an amplitude-modulated signal.*

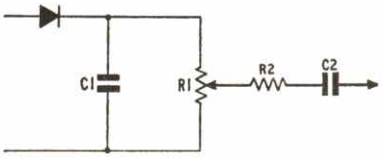
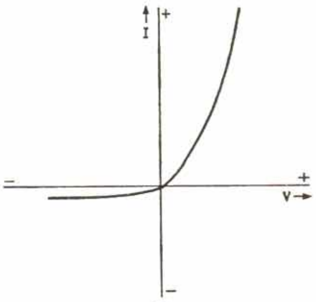


Fig. 2

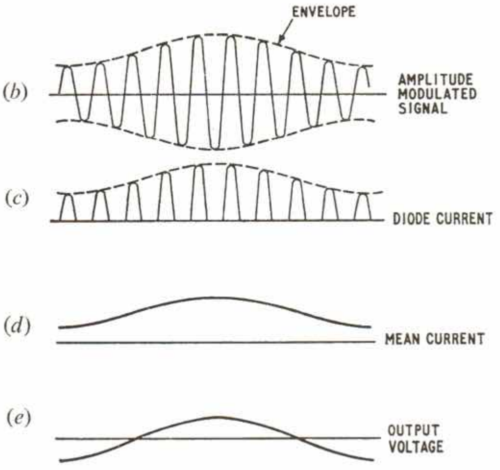
Explain clearly the function of each component, including the diode detector.

State suitable values for C1, C2, R1, and R2.

A. 9. Because of the uni-directional properties of the germanium diode shown in sketch (*a*), if the amplitude-modulated waveform shown in sketch (*b*) is applied to the germanium-diode detector circuit given in Fig. 2 of the question, then, the positive swings of the input signal will cause a pulse of current to flow through the semiconductor diode. These pulses are illustrated in sketch (*c*). By choosing suitable values of C1 and R1 the envelope of these pulses may be made to closely follow the envelope of the original amplitude-modulated signal in sketch (*a*). The voltage across C1 and R1, besides containing the wanted modulation enevelope, also includes a d.c. component and a residual amount of r.f. Employing a large value of C1 largely removes the r.f. component but resistor R1, together with the input capacitor of the following transistor audio-frequency amplifier stage, affords additional r.f. filtering. The capacitor C1 acts as a blocking capacitor to any d.c. component. These last two steps are illustrated in sketches (*d*) and (*e*).



(*a*)

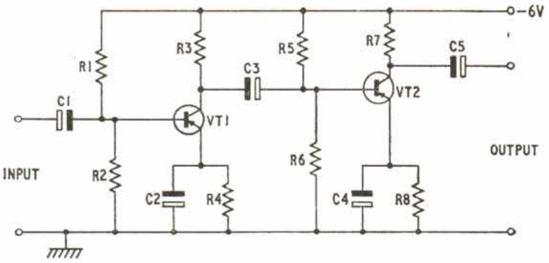


Typical values for C_1 and R_1 are 100 pF and 50 kohms, respectively. The resistance of R_2 needs to have a value of about 5 kohms to provide a high source resistance to the following transistor amplifier stage and C_2 is about 1μF because of the low input impedance of the following transistor amplifying stage.

Q. **10.** *Sketch a circuit and describe the operation of a two-stage RC coupled audio-frequency amplifier using transistors in the common emitter configuration.*

State approximate values of the load resistors and coupling capacitors. Discuss any factors which affect the stage amplification.

A. 10. The sketch shows the circuit diagram of a two-stage RC coupled audio-frequency transistor-amplifier using transistors in the common-emitter configuration. The first stage is biased and d.c. stabilized by the potential divider, R1 and R2 and the emitter resistor, R4. Resistors R5, R6 and R8 perform similar functions for the second stage. For each stage, the base-emitter junction is biased in the forward direction and the collector-base junction is biased in the reverse direction. A signal applied to the input of the first stage will cause an increase in the base current, resulting in an increase in the voltage across the emitter-base junction and a corresponding change in the collector current. The current gain of the common-emitter circuit is given by, $\alpha^1 = \dfrac{\text{change in } I_c}{\text{change in } I_b}$, and typical current gains of 30–50 are possible with this type of configuration. This current gain represents a voltage gain when considering the changing collector current passing through the output resistor R3. This amplified signal voltage is passed via capacitor C3 to the second stage of the amplifier where further amplification will take place.



Common values for the coupling capacitors C_3 and C_5 are 10–25 μF, while typical values for the load resistors R_3 and R_7 are 3·9 kohms and 1·0 kohms, respectively.

The low-frequency response is largely determined by the need to match the low input resistance of the common-emitter transistor which has a value of the order of 1 kilohm. Hence, the values for C_3 and C_5, stated above, will give, at a frequency of 50 c/s, a reactance comparable with the input resistance of the transistor. The response at the higher audio frequencies is largely determined by the self-capacitance of the transistor and the transit time of the majority carriers. These factors are usually incorporated in what is referred to as the cut-off frequency response of the transistor. This is the frequency at which the current gain is reduced to 3 db relative to the gain at the medium audio frequencies.

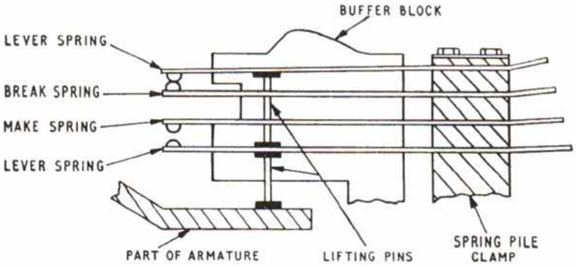
TELEPHONY AND TELEGRAPHY A, 1966

Students were expected to attempt not more than any six questions.

Q. **1.** *Give typical values of contact pressure and contact clearance used on a general purpose telephone relay. Explain with the aid of a sketch how the pressure and clearance are achieved in the design of a springset consisting of one make and one break contact.*

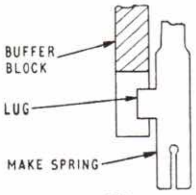
Sketch a graph to show how the load on the armature builds up as the springset is moved from the normal to the fully operated position.

A. 1. The British Post Office 3000-type relay, which may be regarded as a general-purpose type, is designed so that its contact springs are tensioned against the relay armature, or the buffer block, to give pre-determined and stable contact pressures. Sketch (*a*) shows the layout of a springset consisting of one make and one break contact. Both the make and break springs have projecting lugs which enable the spring to be tensioned against the buffer block and sketch (*b*) shows the principle. The tension of the make spring against the buffer block is nominally 18 grammes; the tension of its partner lever spring against the armature (via the lifting pin) is nominally 6 grammes.



(*a*)

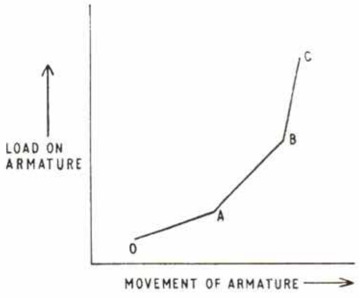
The break spring (with its lever spring lifted away) is tensioned against the buffer block with a pressure of 18 grammes, nominal, while the lever spring is tensioned downwards so that, when resting on the break spring, it needs a force of 6 grammes, nominal, to move the lifting pin from the armature stud. When the armature is normal, the break spring should be held not less than 3 mils away from the buffer block by the pressure of the lever spring. When the armature is operated, the make spring should be lifted clear of the buffer block by at least 3 mils.



(*b*)

The resultant make and break contact pressures are about 20 grammes. The make-contact gap with the relay normal, and the break-contact gap with the relay operated, should not be less than 10 mils, or a clearance of the height of a contact dome. The clearances are catered for in the design by the separation of the contact springs in the build-up of the spring pile, by the degree of armature travel and by the positioning and clearances of the lifting pins. Fine adjustment of contact clearances may be effected by slightly bending the dividing tips of the springs.

An illustration of the way the load on the armature builds up as the relay operates is given in sketch (*c*). Initially, as the armature starts to move, the top lever-spring is lifted with the aid of the break spring and the bottom lever spring is lifted on its own. At point A, the upward movement of the break spring is arrested by the buffer block and the full pressure of the top lever spring bears on the armature. The load therefore increases along A–B. At point B, the bottom lever spring makes contact with the make spring and an additional load results. As final movement of the armature takes place the springs are fully flexed and the load increases to its ultimate value along B–C.



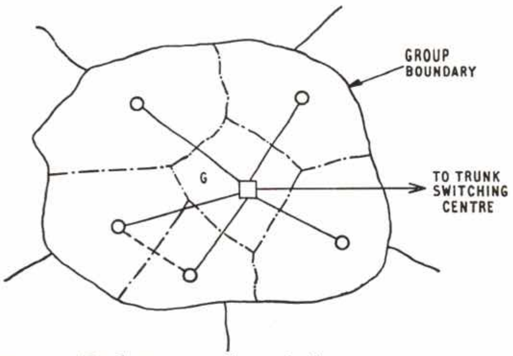
(*c*)

***Q.* 2.** *Give a brief description, with sketches, of a switching plan for setting up telephone calls between any two exchanges in a country-wide network.*

Indicate those links that would normally employ alternating current methods of signalling. Give reasons.

A. 2. The sketches illustrate the principle of a switching plan for a national telephone network over which calls may be set up between any two exchanges. The country should be considered as being divided into a number of large areas, called zones, and each zone into a number of smaller areas called groups. The groups, in turn, are divided into yet smaller areas, each of which is served by a local exchange The main, of trunk switching, centre of a zone switches traffic to and from other zones, and between group switching centres in its own zone. The main or groups witching centre (G.S.C.) of a group switches traffic to and from its zone centre and also acts as a tandem switching centre for local exchanges within its own group.

The routing of various classes of call may now be considered. Local exchanges, to which subscribers are connected, are normally given direct access to each other only when the volume of traffic between them is high enough to justify the cost of line plant provision. To establish a connexion between two local exchanges where direct junctions are not justified, the call is routed via one or more tandem switching centres. If the local exchanges are connected to the same G.S.C., then the G.S.C. will act as the tandem switching centre, as shown in sketch (*a*). If, however, the local exchanges are in adjacent groups, then the most likely routing would be via their home G.S.C.s, as shown in sketch (*b*). Where traffic is insufficient to warrant direct trunk circuits between G.S.C.s, the call would be set up via a trunk switching centre (T.S.C.). All T.S.C.s are fully interconnected so that in the scheme shown, any G.S.C. may be connected to any other G.S.C. over not more than three links. It follows that any two local exchanges may be connected together without using more than five links. Sketch (*c*) shows the overall routing in schematic form.



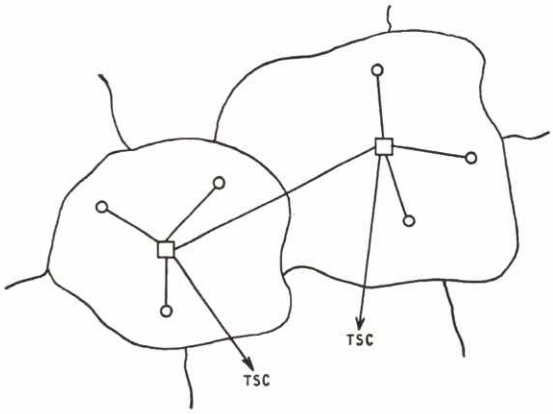
—·—·—· Exchange area boundaries

□ Group switching centre

○ Local exchanges

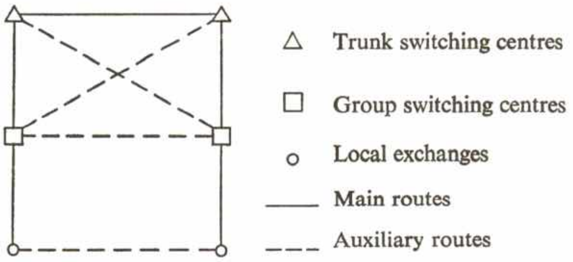
G is an area served by a local exchange associated with a group switching centre

(*a*)



(*b*)

Links interconnecting T.S.C.s would employ alternating-current methods of signalling; so, in many cases, would T.S.C.–G.S.C. and G.S.C.–G.S.C. links. The principal reasons for using this type of signalling arise from the need to transmit digital information, in the form of Strowger pulses, over long distances without serious distortion. For transmission reasons, long-distance circuits require to be amplified. Where a physical path exists (e.g. on audio amplified circuits) it would be possible to transmit direct-current pulses over the phantom. Usually, however, the line would introduce so much distortion that the received pulses would be unusable. The pulses must therefore be transmitted in the same way as speech, i.e. as audio-frequency signals. It is customary nowadays for the great majority of trunk circuits to be provided by means of carrier and coaxial transmission systems. With these, individual channels do not possess physical paths and there is no alternative to voice-frequency signalling.

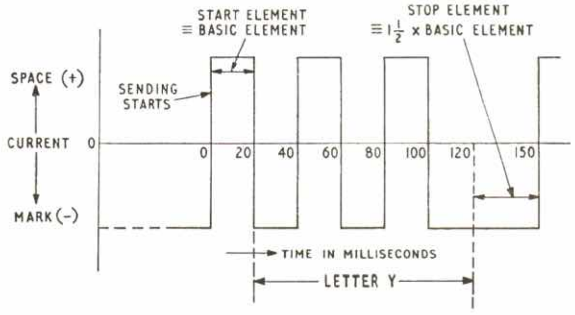


Q. 3. *Describe: (a) the Morse code, (b) the five-unit code.*
Explain why only one of these codes is suitable for teleprinter working.
Sketch a current/time graph of the signal elements comprising a character transmitted from a teleprinter.

A. 3. (*a*) The Morse code is the name given to a system of coding in which numerals, and letters of the alphabet, are represented by various combinations of two basic elements called "dot" and "dash". The number of elements per character is not fixed. For example, the letter "e" is represented by a single dot (·) because it is a commonly-used letter, while the letter "q" is represented by the combination (– – · –). The dash is made three times the length of a dot to enable the two elements to be easily identified. The code is used in serial form, i.e. the elements are sent one after the other, with the spacing between the elements forming a character made the length of a dot, and the spacing between characters the length of three dots. The code may be used to transmit information from one point to another by assigning signals of corresponding lengths to the two basic elements. Thus, we may use long and short lamp flashes, buzzer tones, sounder taps, electrical pulses, etc. It will be seen from this that the speed of sending and receiving information is governed by the signalling method, the type of signalling apparatus used, and the characteristics of the transmission medium.

(*b*) The five-unit code is based on two elements of equal duration the "mark" and the "space". Five such elements in various combinations form a character. There are always five elements in every character, and there is no separation between elements. The maximum number of combinations given by this code is 2^5 or 32.

The five-unit code is particularly suited to teleprinter working because the number of elements per character is constant and this simplifies the design of the transmitting and receiving mechanisms.



When used in this way, each five-unit character is preceded by a "start" signal and followed by a "stop" signal to synchronize the mechanisms of the sending and receiving teleprinters.

The sketch shows the current/time relationship of the elements forming a character, as transmitted from a teleprinter. For the purposes of illustration it is assumed that the signals are being sent into a non-reactive line. In translating the code elements into signals, the mark is represented by a negative current and the space by a positive current.

Q. 4. *Describe the principal functions of the cord circuits at a manual switchboard.*
Assuming that only local calls are handled at a manual exchange what determines the number of operators' positions that need to be staffed and the number of cord circuits that need to be provided at each position?

A. 4. Cord circuits at a manual exchange are the switching links by means of which the operator has complete flexibility in connecting one subscriber to any other subscriber, or to a trunk or junction circuit. The cord circuits are provided in sufficient quantities to cater for the amount of traffic which arises during the busy hour. They provide facilities for the establishment, progress, and cleardown of calls which it would be unnecessary and expensive to associate with each subscriber's line.

The principal functions of a cord circuit are as follows:

(*a*) To provide a connecting path, under the control of the operator, between any two subscribers, or between any subscriber and any other service terminating at the exchange.

(*b*) To provide a transmission bridge whereby transmitter current is fed to each of the two subscribers connected.

(*c*) To provide means whereby the operator can supervise both sides of the connexion during and after the establishment of a call, i.e. by supervisory lamp indications and by speaking and monitoring.

(*d*) To provide means of calling the wanted subscriber, usually by operating a RING key to connect ringing current to the tip and ring of the calling cord.

(*e*) To provide a convenient way of testing to see whether the wanted subscriber is free. The sleeve of a busy subscriber's multiple jack will be at battery potential from the sleeve of another cord circuit. If the operator taps the tip of her calling plug against the jack sleeve, a click will be heard, thus indicating that the wanted subscriber is busy.

(*f*) To provide a means of operating the calling subscriber's meter when the called subscriber answers.

The number of cord circuits and the number of positions which need to be provided, are to some extent interdependent, and are determined by the amount of telephone traffic required to be handled during the busy hour. This, in turn, depends on the number of subscribers connected to the exchange and the busy-hour calling rate. It is reckoned that an individual operator can deal with a maximum of 200 standard calls per hour where a standard call is defined as a call completed at an A-position at a typical C.B. exchange. In assembling an operator's load, an adjustment factor is applied to those calls where the degree of work is, on average, greater or less than that involved in dealing with a standard call. Thus, at certain types of B-position, where there is less work per call, the actual number of calls dealt with during the busy hour may be as high as 500.

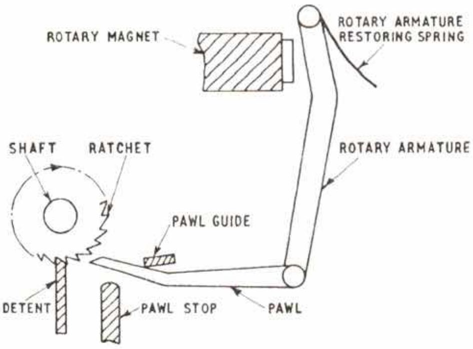
From the foregoing considerations, and taking into account the number of subscribers and the calling rate, the number of positions required to handle the busy-hour traffic is determined. Subscribers with various calling rates have their calling appearances intermingled and distributed over the positions in such a way that, as far as possible, the operators are equally loaded.

The number of cord circuits per position is based on the average number of simultaneous connexions expected from the subscribers served by the position. If the 200 calls per busy hour at an A-position have an average holding time of 3 minutes per call, then on average, $200 \times 3/60 = 10$ simultaneous connexions would be held during the busy hour. If the average holding time were 6 minutes, there would be 20 simultaneous connexions. In practice, on an A-position, 17 cord circuits are provided.

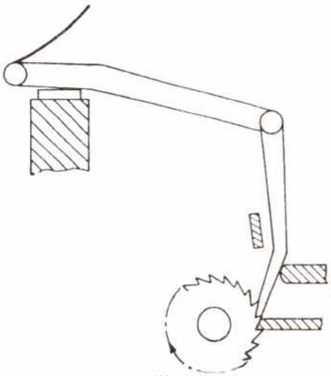
At times outside the busy hour, traffic diminishes and fewer positions are staffed. In these circumstances, an operator makes up her load by taking calls in addition to those originating at her own position.

Q. 5. *Describe with the aid of simple sketches the mechanical principles of the rotary stepping mechanism of a two-motion selector.*
What is the relationship between the spacing of the rachet teeth and the spacing of the bank contacts?

A. 5. Sketches (*a*) and (*b*) show the main components involved in the rotarystepping action of a two-motion selector. Sketch (*a*) shows the situation with the rotary magnet normal, while sketch (*b*) shows the operated conditions. The mechanism operates on a forward-acting principle, i.e. the selector shaft, which carries the wipers, moves upon operation of the rotary magnet, in contrast to a reverse-acting mechanism which moves upon release of the magnet.

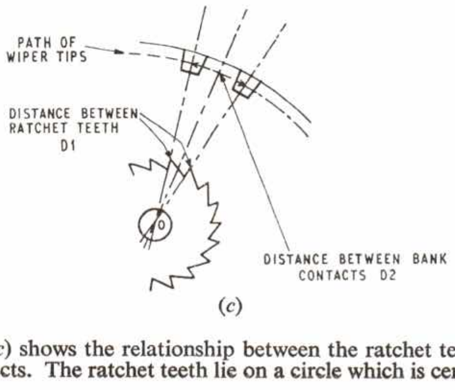


(*a*)



(*b*)

The tip of the rotary pawl is normally held clear of the rotary ratchet under the influence of the pawl guide and the rotary armature restoring-spring. When the armature starts to move towards the rotary magnet, upon energization of the latter, the rotary pawl moves towards the rotary ratchet and the pawl tip engages with the long face of a ratchet tooth. As the armature and pawl movements continue, the pawl tip is guided by the pawl guide into the root of the ratchet tooth. Finally, when the armature stroke is complete, the pawl tip pushes the tooth so that the ratchet rotates one step, thus moving the wipers from one contact to the next. The pawl stop prevents overshoot. Just as the ratchet takes up its new position, the rotary detent drops into place behind the next forward tooth, thus preventing the shaft from moving back under the action of the shaft restoring-spring (not shown) when the pawl withdraws. The selector circuit arrangements are such that the rotary magnet is de-energized immediately before the armature reaches the end of its forward stroke.


PATH OF WIPER TIPS
DISTANCE BETWEEN RATCHET TEETH D1
O
DISTANCE BETWEEN BANK CONTACTS D2

(*c*)

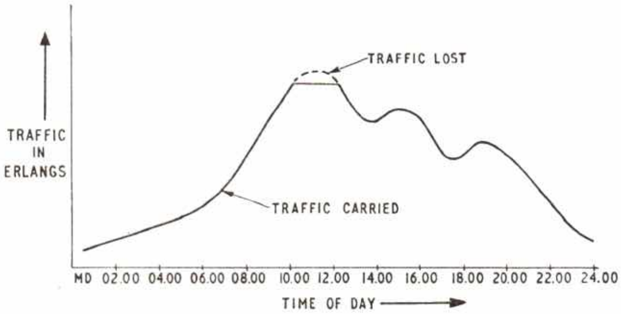
Sketch (*c*) shows the relationship between the ratchet teeth and the bank contacts. The ratchet teeth lie on a circle which is centred on the selector shaft; the bank contacts lie on an arc which is, in effect, also centred on the selector shaft. The ratchet teeth and bank contact therefore subtend the same angle at the centre of the shaft and the spacings are related as follows:

$$\frac{\text{D1}}{\text{D2}} = \frac{\text{Distance from shaft centre to ratchet teeth}}{\text{Distance from shaft centre to bank contacts}}.$$

Q. **6.** *Sketch a graph to illustrate variations of telephone traffic at an automatic exchange during a typical 24-hour period. Mark on your graph the traffic that is carried and that which is allowed to fail.*

A particular group of trunks can be offered 12 *erlangs of traffic during the busy-hour. What determines the number of calls that may be offered to this group during the busy-hour? Give numerical examples.*

A. 6. The sketch illustrates, in idealized form, the incidence of telephone traffic which might arise at a typical medium-size town.



The main busy-hour usually lasts from about 10 30 hrs to 11 30 hrs, and it is during this period that most traffic is offered (and carried) and most is lost. A slight amount of traffic is lost each side of the busy hour, but apart from this the loss is negligible. There is a second, but lower, peak of traffic during the afternoon, and another, still lower, during the early evening.

The traffic flow (in Erlangs) for a period is the number of calls originated in the period multiplied by the average holding time of a call expressed in terms of the period.

Thus, one call lasting for one hour is effectively the same amount of traffic as 60 calls lasting, on average, $\frac{1}{60}$ hr (one minute). In the same way 12 erlangs of traffic will be generated by 12 calls lasting for an average of one hour, or 48 calls lasting on average $\frac{1}{4}$ hr (15 minutes), or 144 calls lasting on average $\frac{1}{12}$ hr 5 minutes, or 240 calls lasting on average $\frac{1}{20}$ hr (3 minutes).

As the question refers to traffic offered and calls offered, no allowance need be made for calls lost (due to insufficiency of plant) during the busy hour.

Q. **7.** *What are the functions of the main and intermediate distribution frames?*

Sketch a simple trunking diagram of a 10,000-*line automatic telephone exchange and show how a local call is routed via these frames.*

A. 7. The main distribution frame (M.D.F.) is primarily a major termination point for the external lines on the one hand and the internal switching system on the other. External cables and distribution points are terminated in appropriate order on one side of the M.D.F., while the subscribers' exchange connexions are terminated in exchange number order on the other. By the use of semi-permanent wiring, known as "jumpers", any cable pair can be cross-connected to any exchange number with complete flexibility. External cable-pairs can also be interconnected (e.g. on through circuits) and junction and trunk circuits can be cross-connected to internal tie-cables leading to the appropriate exchange equipment.

As the boundary of the exchange, the M.D.F. is a convenient and suitable point at which to provide protective devices, such as fuses, which guard the exchange equipment from the excess voltages and currents which sometimes arise on external lines due to fault and storm conditions.

Facilities are provided on the M.D.F. for intercepting subscribers' lines for testing purposes. At some exchanges, service tones can be connected to lines at the M.D.F. by means of flexible cords.

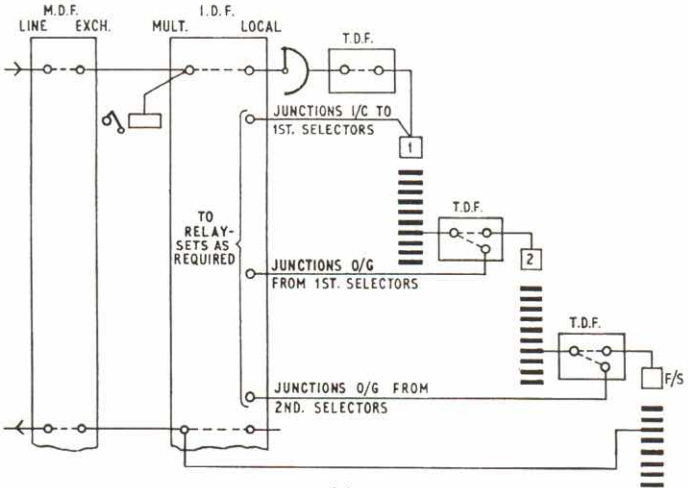
The intermediate distribution frame (I.D.F.) is the point where various items of exchange equipment are terminated and where subscribers' lines, incoming from the M.D.F. in exchange number order, are cross-connected to individual calling equipment. Again, full flexibility is afforded so that subscribers, according to their calling rates, may be so connected to the equipment that the traffic load is spread evenly throughout the exchange.

The subscribers' multiple is terminated at the I.D.F., and it is here also that the subscribers' meters are connected and the P-wire is introduced.

Various interception and observation services terminate on the I.D.F. and are connected to individual subscribers' multiple appearances as required.

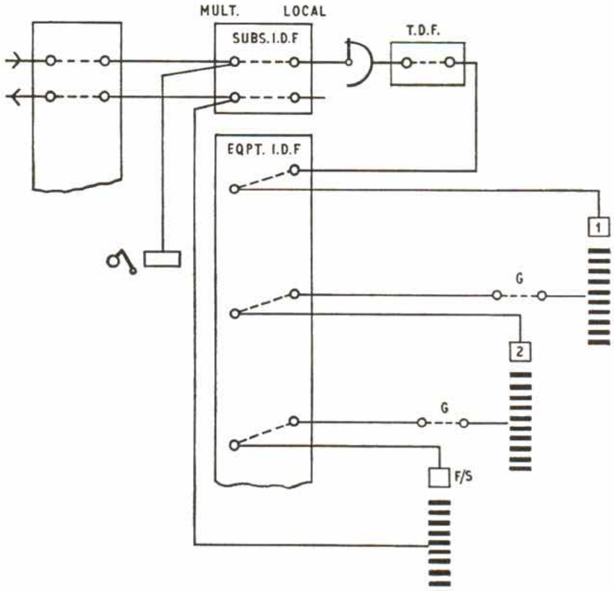
The exchange side of the M.D.F. is connected to the multiple side of the I.D.F. by internal tie-cables; the external lines are connected to the line side of the M.D.F., while the subscribers' calling equipment is terminated on the local side of the I.D.F.

The sketches show the trunking of typical non-director exchanges, each of 10,000 lines, but having different cabling arrangements. It is assumed in both cases that all the lines are available for subscribers and only two numerical-stages have therefore been shown. Actually, several levels of the first selector would no doubt be utilized to give access to subscriber trunk dialling (S.T.D.) registers and code-dialled junctions, so that another switching stage would be necessary to cater for a subscriber's multiple capacity of 10,000 lines.


M.D.F.
LINE
EXCH.
I.D.F.
MULT.
LOCAL
T.D.F.
JUNCTIONS I/C TO 1ST. SELECTORS
1
T.D.F.
TO RELAY-SETS AS REQUIRED
JUNCTIONS O/G FROM 1ST. SELECTORS
2
T.D.F.
F/S
JUNCTIONS O/G FROM 2ND. SELECTORS

(*a*)

Sketch (*a*) illustrates the arrangements at an exchange where grading between stages is carried out on separate trunk distribution frames (T.D.F.). Provision is made for cabling certain selector outlets back to the I.D.F. for connexion to outgoing junction equipment, as required. Similarly, incoming junction equipment can be connected to incoming first selectors.



(*b*)

In sketch (*b*) there are, effectively, two I.D.F.s, a local or subscribers' I.D.F. and an equipment I.D.F. grading between stages is carried out at the rear of the selector racks as indicated at G, and cross-connexion made at the equipment I.D.F.

Q. **8.** *Design a grading for* 20 *trunks from one level of* 100*-outlet two-motion selectors and show how the optimum arrangement is achieved. Explain why a grading is unnecessary when full-availability conditions exist.*

A. 8. There are 10 outlets from each level of a 100-outlet two-motion selector, i.e., the availability is 10.

The number of grading groups is given by the formula:

Number of groups $= \frac{2N}{A}$ where N is the number of trunks, and A is the availability.

In this case, $N = 20$ and $A = 10$, so that the number of groups

$$= \frac{2 \times 20}{10} = 4.$$

With a 4-group grading, to preserve uniformity, there will be singles, doubles and full commons, but no triples.

Let a be the number of singles, b the number of doubles, and c the number of full commons.

$$\text{Then, } 4a + 2b + c = 20,$$
$$\text{and } \quad a + b + c = 10.$$

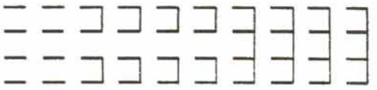
$$\text{Subtracting gives, } 3a + b = 10.$$

Substituting various values for a we can draw up a table for corresponding values of b and c:

$$\begin{array}{lrrr} & a = & 1 & 2 & 3 \\ & b = & 7 & 4 & 1 \\ & c = & 2 & 4 & 6 \\ \text{Sum of successive differences} & = & 11 & 2 & 7 \end{array}$$

The best grading arrangement is the one in which there is a smooth progression from the singles through to the commons, i.e. where the sum of successive differences is least.

In this instance, the preferred grading is $a = 2$, $b = 4$, $c = 4$ and the layout is shown in the sketch.



Full-availability conditions exist when the number of trunks required to handle the traffic leaving a particular switching stage does not exceed the number of outlets on the selector levels at that stage. Thus, in the example above, if 9 trunks were required instead of 20, all the trunks would be fully available to all groups and there would be no need to share the trunks among the early choices of each group in order to distribute 20 trunks over 10-outlet selectors.

Q. **9.** *What purposes are served by the main battery at an automatic telephone exchange? Briefly compare the merits of charge–discharge and floated-battery working. Which system requires a means of automatic voltage-control and why?*

A. 9. The principal function of the main battery at an automatic telephone exchange is to supply power to operate the switching and signalling equipment and to feed current to the subscribers' telephones. In the case of charge–discharge working, where one battery serves the exchange while the other is being charged, the discharging battery supplies its power direct. With the floated battery system, where the charging source is joined across the battery and both are connected to the exchange load, the battery acts as a reservoir to smooth out ripple and fluctuations emanating from the charging source; it also acts as a standby in the event of failure of the charging rectifier or generator.

The low-impedance property of the battery is also useful in helping to decouple the various speech, switching and other circuits which are working from the common supply.

With the charge–discharge system, the two batteries are alternately fully charged and discharged. The power-conversion efficiency of this arrangement is not very high (about 80 per cent). Another disadvantage is that the continuous charge–discharge cycling tends to loosen the active material on the battery plates and hence shorten the life of the battery. The system also suffers from the drawback that, if prolonged mains failure occurred, a considerable amount of manipulation would subsequently be necessary to restore one of the batteries to a fully-charged condition. Even under normal circumstances, fairly frequent attention is called for in changing-over the batteries and setting the charge.

With floated-battery working, on the other hand, the battery is maintained in a state of full charge, or near full charge, at all times and does not normally undergo the charge–discharge cycle. This considerably prolongs the life of the battery, and ensures that a considerable power reserve is always available. when after a prolonged failure, the mains supply is restored the rectifier or generator supplies excess current to re-charge the battery while at the same time supplying the load. Floated-battery working generally requires less manual attention. One feature of the system is that, as the floated battery is rarely fully discharged, its residual capacity is somewhat less than that of a battery of the same nominal capacity which has been subjected to a series of charge–discharge cycles.

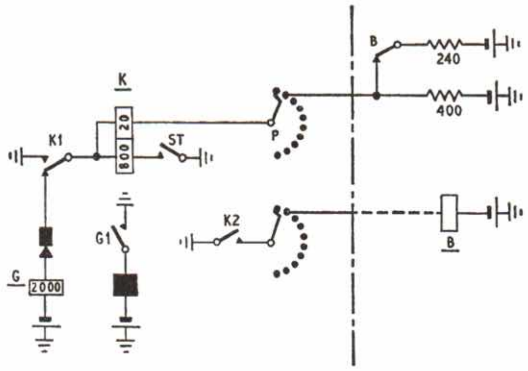
A voltage-control system is necessary with floated-battery working because, under light-load conditions, the continual charging current, although controlled to a low value, causes the battery voltage to rise above the upper limit to which the exchange equipment was designed to work. One method of control is to switch in counter-e.m.f. cells in opposition to the main battery to reduce the supply voltage to a value below the upper limit. When, under increased load conditions, the supply voltage falls to the lower design limit, the counter-e.m.f. cells are switched out to allow the resultant voltage to rise to within the upper and lower limits.

Q. **10.** *Sketch and describe a circuit element for controlling the hunting and testing action of a selector using battery testing.*

What is the approximate operating time of the drive-cutting relay? What happens if this operating time becomes excessive?

A. 10. The sketch shows a circuit element for controlling the hunting and testing action of a uniselector using battery testing; the arrangements for guarding a seized outlet are also shown.

The sequence is started by the operation of contact ST. A circuit is completed for relay G via the 800-ohm coil of relay K. The characteristics of the relays are such that, in these circumstances, relay G operates but relay K does not. Contact G1 completes a circuit for the uniselector drive-magnet, which operates. The operation of the drive-magnet in turn operates the drive-magnet interruptor contacts, and this disconnects the circuit for relay G. In releasing, contact G1 disconnects the drive-magnet circuit and the armature restores, moving the uniselector wipers to the first working outlet. If the first outlet is busy, as indicated by a disconnexion or the presence of an earth condition on the bank contact, relay K remains unoperated and the sequence repeats itself.



This continues, and the wipers are stepped around the bank, until a free outlet is found. The free condition is a battery potential applied to the bank contact through two parallel resistors giving an effective resistance of 150 ohms. In conjunction with the 20-ohm coil of relay K, this resistance battery shunts the current through relay G to a value below its operate point. Relay G does not therefore re-operate and re-energization of the drive-magnet is prevented. Relay K now operates over its two coils in series and disconnects relay G. Contact K1 short-circuits its 800-ohm coil, but the relay holds over its 20-ohm coil. In so doing, it also guards the bank outlet against seizure by another uniselector by reducing the effective battery potential, as seen at the bank contact, to a low value. This is subsequently further reduced when the operation of relay B in the seized selector disconnects the 240-ohm resistor from the parallel arrangement, thus increasing the battery resistance.

The operate time of relay K is approximately 10 milliseconds. If the operate time became excessive the uniselector would stop on the free outlet, as before, because of the non-operation of relay G, but guarding of the outlet (by contact K1) would be delayed and hence the risk of a double connexion would be correspondingly increased.

MATHEMATICS B, 1966

Students were expected to attempt any six questions.

Q. **1.** (*a*) *Solve for* x *and* y *the equations* $3y - 4x + 1 = 0$, $6x^2 + xy = 3(y^2 + 1)$.

(*b*) *By rearranging the function* $y = 3x^2 - 14x + 18$ *in the form* $y = a(x + b)^2 + c$, *where* a, b *and* c *are constants, show that* y *cannot be less than* $1\frac{2}{3}$.

What is the value of x, *and of* $\frac{dy}{dx}$, *when* $y = 1\frac{2}{3}$?

A. 1. (*a*)

$$3y - 4x + 1 = 0,$$
$$6x^2 + xy = 3(y^2 + 1).$$

From the first equation, $y = \dfrac{4x - 1}{3}$

Substituting for y in the second equation,

$$6x^2 + \frac{x(4x - 1)}{3} = 3\left\{\frac{(4x - 1)^2}{9} + 1\right\}$$

Multiply throughout by 3 and re-arrange as follows:

$$18x^2 + 4x^2 - x - (4x - 1)^2 - 9 = 0,$$
$$\text{or,}\quad 22x^2 - x - 16x^2 + 8x - 1 - 9 = 0.$$
$$\therefore\quad 6x^2 + 7x - 10 = 0,$$
$$\text{or,}\quad (6x - 5)(x + 2) = 0.$$
$$\therefore\quad x = \frac{5}{6} \text{ or } -2,$$
$$\text{whence,}\quad y = \frac{7}{9} \text{ or } -3.$$

Thus $\underline{x = \frac{5}{6}, y = \frac{7}{9}}$ or, $\underline{x = -2, y = -3.}$

Note. The solutions should be checked in the original equations.

(*b*) $$y = 3x^2 - 14x + 18.$$

If y is arranged in the form $a(x + b)^2 + c$, then ,

$$y = ax^2 + 2abx + ab^2 + c.$$

For this form to be identical with the original form,

$$ax^2 = 3x^2, \quad\text{or}\quad a = 3,$$
$$2abx = -14x, \quad\text{or}\quad ab = -7,$$
$$\text{and}\quad ab^2 + c = 18.$$

Hence, substituting $b = -\frac{7}{3}$ in the last equation gives:

$$3\left(-\frac{7}{3}\right)^2 + c = 18,$$
$$\text{or,}\quad c = 18 - \frac{49}{3} = \frac{5}{3}.$$

Thus, in the form required,

$$y = 3\left(x - \frac{7}{3}\right)^2 + \frac{5}{3}.$$

As $\frac{5}{3}$ is constant, the minimum value of y will occur when $3\left(x - \frac{7}{3}\right)^2$ is zero, as, being a square, this latter term cannot become negative.

$$\therefore\quad y_{\text{minimum}} = 0 + \frac{5}{3} = 1\tfrac{2}{3}. \qquad \text{Q.E.D.}$$

When $y = 1\frac{2}{3}$,

$$3\left(x - \frac{7}{3}\right)^2 = 0.$$
$$\therefore\quad x = \frac{7}{3} = \underline{2\tfrac{1}{3}}.$$

$$\frac{dy}{dx} = 6x - 14.$$

When $x = 2\tfrac{1}{3}$, $\dfrac{dy}{dx} = 6\left(\dfrac{7}{3}\right) - 14 = 0.$

Thus, when $y = 1\tfrac{2}{3}$, $\underline{\dfrac{dy}{dx} = 0.}$

Q. 2. *(a) If* $\log_{10}(10 + 9x) - \log_{10}(11 - x) = 2$, *find* x.

(b) Prove that $\dfrac{\log_2 N}{\log_3 N}$ *is independent of the number* N, *and calculate its value to three significant figures.*

(c) A system of n *receivers is said to have a diversity gain of* G *decibels given by*

$$G = \frac{20}{m}\left(1 - \frac{1}{n}\right)\log_e\left(\frac{1}{k}\right),$$

where k *is the proportion of time loss allowed, and* m *is a constant for the system.*

(i) If $m = 2{\cdot}1$ *when* $n = 3$ *and* $k = 0{\cdot}2$, *calculate* G.

(ii) Express k *in terms of* G, m *and* n.

A. 2. (*a*) $\log_{10}(10 + 9x) - \log_{10}(11 - x) = 2.$

If $\dfrac{a}{b} = k$, then $\log_{10}a - \log_{10}b = \log_{10}k$.

Thus, by comparison with the given equation,

$$\frac{10 + 9x}{11 - x} = \text{antilog } 2 = 100.$$

$$\therefore\quad 10 + 9x = 100(11 - x),$$
$$= 1{,}100 - 100x.$$
$$\text{Or,}\quad 109x = 1{,}090,$$
$$\text{or,}\quad x = \underline{10.}$$

(*b*) $\dfrac{\log_2 N}{\log_3 N}$.

Let $\log_2 N = a$, and $\log_3 N = b$.

Then $N = 2^a$ and $N = 3^b$, by definition of a logarithm.

Taking logarithms of N to base 10,

$$\log_{10}N = a\log_{10}2 = b\log_{10}3.$$

$$\therefore\quad \frac{a}{b} = \frac{\log_2 N}{\log_3 N} = \frac{\log_{10}3}{\log_{10}2},\ \text{which is independent of } N.$$

Q.E.D.

$$\frac{a}{b} = \frac{0{\cdot}4771}{0{\cdot}3010},$$
$$= 1{\cdot}585,$$

No.	*Log.*
4·771	0·6786
3·010	0·4786
	0·2000

$$\therefore\quad \frac{\log_2 N}{\log_3 N} = \underline{1{\cdot}59}\ \text{to three significant figures.}$$

(*c*)
$$G = \frac{20}{m}\left(1 - \frac{1}{n}\right)\log_e\left(\frac{1}{k}\right).$$

(*i*) When $m = 2{\cdot}1$, $n = 3$ and $k = 0{\cdot}2$,

$$G = \frac{20}{2{\cdot}1}\left(1 - \frac{1}{3}\right)\log_e\left(\frac{1}{0{\cdot}2}\right),$$
$$= \frac{40}{6{\cdot}3}\log_e 5,$$
$$= \frac{40}{6{\cdot}3}\,1{\cdot}6094,$$
$$= \underline{10{\cdot}22.}$$

No.	*Log.*
40	1·6021
1·6094	0·2065 +
	1·8086
6·3	0·7993 —
	1·0093

(*ii*)
$$\log_e\left(\frac{1}{k}\right) = \frac{mG}{20}\,\frac{1}{1 - \dfrac{1}{n}},$$
$$= \frac{mnG}{20(n - 1)}$$

Since $\log_e k = -\log_e\left(\dfrac{1}{k}\right)$,

then, $\log_e k = \dfrac{mnG}{20(1 - n)}$.

$$\therefore\quad \underline{k = e^{\frac{mnG}{20(1 - n)}}.}$$

Q. 3. *State the expansions of* $\cos(A + B)$ *and* $\sin(A + B)$ *in terms of the sines and cosines of* A *and* B.

Without using tables, show that $\cos 75° = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$.

If $\cos\phi = 0{\cdot}8$, *and* ϕ *is acute, express* $35\sin(\omega t - \phi)$ *in the form* $a\sin\omega t + b\cos\omega t$.

Sketch the graph of $y = 35\sin(\omega t - \phi)$ *from* $t = -\dfrac{2\pi}{\omega}$ *to* $t = +\dfrac{2\pi}{\omega}$.

A. 3.
$$\cos(A + B) = \underline{\cos A\cos B - \sin A\sin B.}$$
$$\sin(A + B) = \underline{\sin A\cos B + \cos A\sin B.}$$

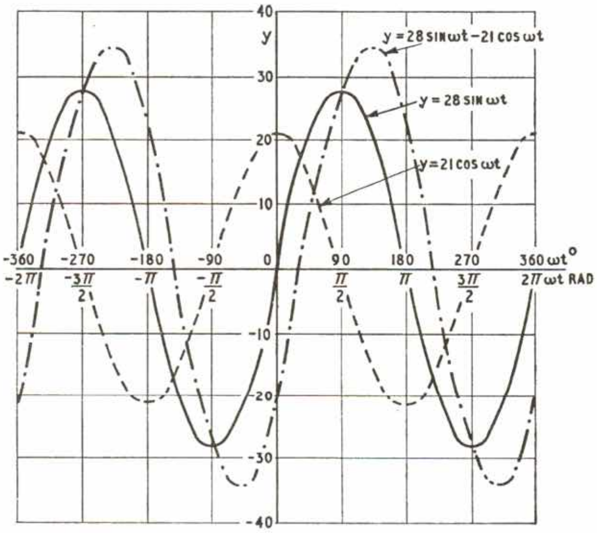
$$\cos 75° = \cos(45° + 30°),$$
$$= \cos 45°\cos 30° - \sin 45°\sin 30°,$$
$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right),$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}.$$

Q.E.D.

$$35\sin(\omega t - \phi) = 35\{\sin\omega t\cos\phi - \cos\omega t\sin\phi\}.$$
$$\sin^2\phi + \cos^2\phi = 1.$$
$$\therefore\quad \sin^2\phi = 1 - \cos^2\phi = 1 - 0{\cdot}64 = 0{\cdot}36.$$

$\therefore\quad \sin\phi = 0{\cdot}6$ (as ϕ is acute, $\sin\phi = -0{\cdot}6$ is inadmissible).

$$\therefore\quad 35\sin(\omega t - \phi) = 35(0{\cdot}8\sin\omega t) - 35(0{\cdot}6\cos\omega t).$$
$$= \underline{28\sin\omega t - 21\cos\omega t.}$$



The graph of $35\sin(\omega t - \phi)$ is shown in the sketch. As t varies from $-\dfrac{2\pi}{\omega}$ to $+\dfrac{2\pi}{\omega}$, ωt varies from -2π to $+2\pi$ radians, i.e. from $-360°$ to $360°$. The graphs of $28\sin\omega t$ and $21\cos\omega t$ are also shown, the graph of $y = 35\sin(\omega t - \phi)$ being obtained as the difference between them.

Q. 4. *(a) Each year the capital value of a computer is reckoned to depreciate by 15 per cent of its value at the beginning of that year. In how many years will its capital value be first below 10 per cent of its original value?*

(b) State without proof the condition for the geometric progression $a + ar + ar^2 + \ldots$ *to have a sum to infinity.*

Write the recurring decimal $7{\cdot}\dot{4}\dot{5}$ *(i.e.* $7{\cdot}454545\ldots$*) in terms of an infinite geometric progression and hence express it as a vulgar fraction.*

A. 4. (*a*) Let A be the initial capital value of the computer. After the first year, the depreciated value will be $100 - 15 = 85$ per cent of A.

$\therefore$ After 1 year the value $= 0{\cdot}85A$.

$$\text{After 2 years the value} = 0{\cdot}85A - \frac{15}{100} \times 0{\cdot}85A,$$

$$= 0{\cdot}85A(1 - 0{\cdot}15) = 0{\cdot}85^2A.$$

$$\text{After 3 years the value} = 0{\cdot}85^2A - \frac{15}{100} \times 0{\cdot}85^2A,$$

$$= 0{\cdot}85^2A(1 - 0{\cdot}15) = 0{\cdot}85^3A.$$

Hence it is clear that, after n years, the value will be $0{\cdot}85^nA$.

When the capital value becomes equal to 10 per cent of the original value,

$$0{\cdot}85^nA = 0{\cdot}1A.$$

$$\therefore \quad n \log_{10}0{\cdot}85 = \log_{10}0{\cdot}1,$$

$$\text{or,} \quad n = \frac{\bar{1}}{\bar{1}{\cdot}9294},$$

$$= \frac{-1}{-0{\cdot}0706},$$

$$= 14{\cdot}17.$$

No.	*Log.*
1	$0{\cdot}0000$
$0{\cdot}0706$	$\bar{2}{\cdot}8488$
	$1{\cdot}1512$

Thus, after 14 years the value will still be slightly above 10 per cent of A and will first be less than this after $\underline{\text{15 years}}$.

(*b*) The condition is that r is numerically less than 1, in which case the sum to infinity is $\dfrac{a}{1-r}$.

$$7{\cdot}4545\ldots = 7 + 0{\cdot}45 + 0{\cdot}0045 + 0{\cdot}000045 + \ldots$$

$$= 7 + 0{\cdot}45 + \frac{0{\cdot}45}{100} + \frac{0{\cdot}45}{10{,}000} + \ldots$$

The fractional part of the number thus forms a geometric progression with a first term 0·45 and a common ratio of $\dfrac{1}{100}$ or 0·01. Since this is less than unity, the sum to infinity of the progression will be,

$$\frac{a}{1-r} = \frac{0{\cdot}45}{1 - 0{\cdot}01} = \frac{0{\cdot}45}{0{\cdot}99} = \frac{5}{11},$$

$$\text{Hence } 7{\cdot}\dot{4}\dot{5} = \underline{7\tfrac{5}{11}}.$$

Q. 5. *The tension T lb wt in a wire rope hauling a cable varies with the distance x feet pulled from rest as follows:*

x	0	20	40	60	80	100
T	300	278	249	209	161	102

Assuming a formula $300 - T = ax + bx^2$ *to relate these figures, plot suitable variables to obtain a straight-line graph, and hence obtain estimates of the constants a and b.*

A. 5.

$$300 - T = ax + bx^2.$$

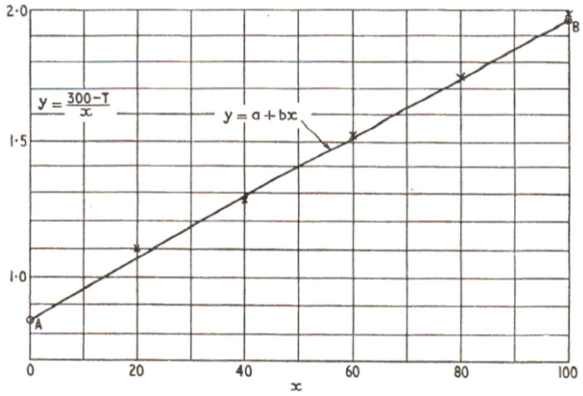
$$\therefore \quad \frac{300 - T}{x} = a + bx,$$

$$\text{or} \quad y = a + bx$$

where $y = \dfrac{300 - T}{x}$ is a new variable dependent on both T and x. As $y = a + bx$ is of linear form, a straight-line graph will result when y is plotted against x, assuming that the given data are related by the formula $300 - T = ax + bx^2$. The required variable is obtained in the following table.

x	0	20	40	60	80	100
T	300	278	249	209	161	102
$300 - T$	0	22	51	91	139	198
$y = \dfrac{300 - T}{x}$	$\dfrac{0}{0}$	1·1	1·275	$1{\cdot}51\dot{6}$	1·7375	1·98

Note.—When $x = 0$, $\dfrac{300 - T}{x} = \dfrac{0}{0}$ as shown in the table; this is known as an indeterminate form and should be disregarded when plotting the graph.



The graph is shown in the sketch, from which it is seen that the points lie approximately on a straight line and hence it may be assumed that the given data are approximately related by the law $y = a + bx$.

From the graph, when $x = 0$,

$$y = 0{\cdot}85 \quad \text{and hence } \underline{a = 0{\cdot}85}.$$

The gradient of the line gives the value of b and may be determined from the coordinates of any two points on the straight line. Taking points A and B (see sketch),

$$b = \frac{1{\cdot}96 - 0{\cdot}85}{100 - 0},$$

$$= \frac{1{\cdot}11}{100} = 0{\cdot}0111,$$

$$\text{Thus,} \quad \underline{b = 0{\cdot}0111}.$$

Q. 6. *A shore radar station A locates a vessel X in distress on a bearing of* 292° *at a distance of* 11,600 *yd. The nearest lifeboat station is at B,* 8,700 *yd from A on a bearing of* 237° (*All bearings measured East of North*).

Calculate the distance and bearing of X from B.

A. 6. $\underline{\text{Bearing of } X \text{ from } B = 339^\circ\ 6' \text{ East of North}}$

$$\underline{XB = 9{,}721 \text{ yds.}}$$

Q. 7. *An aircraft heading due South at* 450 *mile/h encounters a wind of* 120 *mile/h coming from the south-east. Find graphically or by calculation the actual speed and direction of flight of the aircraft.*

Under the same wind conditions, in what direction should the aircraft be headed to maintain a course due South?

A. 7. The vector diagram is shown in the sketch, where AB is a north-south line and the direction DAC is north-westerly, i.e. the direction towards which the wind blows.

If AB represents the aircraft speed of 450 mile/h and AC represents the wind speed, each speed being represented in magnitude and direction, then AR, the resultant vector of AC and AB, will represent the actual speed and direction of flight of the aircraft. AR is the diagonal of the parallelogram formed with AC and AB as adjacent sides.

In triangle ARB, since RB = AC, RB and AB represent magnitudes of 120 mile/h and 450 mile/h, respectively, and $\angle$ ABR = 45°, since $\angle$ DAB = 45°.

Hence, using the cosine rule,

$$\text{AR}^2 = \text{RB}^2 + \text{AB}^2 - 2\,\text{RB.AB} \cos \angle\,\text{ABR}$$

$$= 120^2 + 450^2 - 2 \times 120 \times 450 \cos 45^\circ,$$

$$= 14{,}400 + 202{,}500 - 108{,}000 \times 0{\cdot}7071,$$

$$= 216{,}900 - 76{,}370,$$

$$= 140{,}530.$$

$$\therefore \quad \text{AR} = 374{\cdot}9 \quad \text{from a table of square roots.}$$

No.	*Log.*
108·000	$\underline{5}{\cdot}0334$
0·7071	$\bar{1}{\cdot}8495$
	4·8829

From the sine rule,

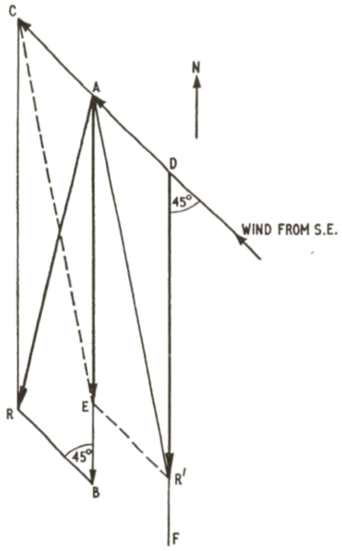
$$\frac{\sin \angle \mathrm{RAB}}{\mathrm{RB}} = \frac{\sin \angle \mathrm{ABR}}{\mathrm{AR}}.$$

$$\therefore \quad \sin \angle \mathrm{RAB} = \frac{120}{374{\cdot}9} \times \sin 45^\circ.$$

$$\therefore \quad \angle \mathrm{RAB} = 13^\circ\, 5'.$$

No.	*Log.*	
120	2·0792	
sin 45°	$\bar{1}$·8495	
	1·9287	+
374·9	2·5739	
	$\bar{1}$·3548	−

Hence the speed of the aircraft is 375 mile/h and it is flying in a direction 13° 5′ East of South.



Assuming the same wind conditions and assuming also the same air speed of 450 mile/h (this was not stated in the question but was perhaps implied), it is clear that the aircraft must be headed somewhat east of south. Referring to the same sketch and taking D as a new reference point, DA represents the wind speed in magnitude and direction. Then if DF is drawn parallel to AB to represent the southerly direction, the vector AR′ is drawn with centre A and radius AB representing 450 mile/h to meet DF in R′. Then DR′ is the resultant of DA and AR′ and thus represents the true ground speed. This may be clearer from the parallelogram ACER′, in which, as before, AC represents the wind speed. AR′ represents the air speed in the second case and hence AE (= DR′) represents the resultant ground speed.

In triangle ADR′,

$$\frac{\sin \angle \mathrm{AR'D}}{\mathrm{AD}} = \frac{\sin \angle \mathrm{ADR'}}{\mathrm{AR'}}.$$

$$\therefore \quad \sin \angle \mathrm{AR'D} = \frac{120}{450} \times \sin 135^\circ$$

$$\angle \mathrm{AR'D} = 10^\circ\, 52'$$

$$\text{But} \quad \angle \mathrm{AR'D} = \angle \mathrm{EAR'}.$$

No.	*Log.*	
120	2·0792	
sin 135°	$\bar{1}$·8495	
	1·9287	+
450	2·6532	
	$\bar{1}$·2755	−

Hence, the aircraft must be headed 10° 52′ East of South in order to maintain a course due South.

Note.—Both parts of this question can be solved graphically by a scale drawing substantially that shown in the sketch.

***Q.* 8.** *(a) P is the point (2, 8) on the curve $y = 3x^2 - 8/x$, and Q is a neighbouring point on the curve, where $x = 2{\cdot}001$. Calculate to three decimal places the slope of the line PQ.*

Repeat the same procedure for the point R on the curve, where $x = 2{\cdot}00001$, giving the slope of PR to five decimal places. What do you infer to be the slope of the tangent at P to the curve?

(b) Differentiate from first principles the function $y = 3x^2 - 8/x$, and sketch its graph, including both positive and negative values of x.

A. 8. (*a*)

$$y = 3x^2 - \frac{8}{x}.$$

When $x = 2$, $y = 8$ these being the coordinates of point P.

When $x = 2{\cdot}001$,

$$y = 3 \times 2{\cdot}001^2 - \frac{8}{2{\cdot}001},$$

$$\simeq 3 \times 4{\cdot}004 - \frac{8}{2(1 + 0{\cdot}0005)}.$$

The first term may be approximated to $3 \times 4{\cdot}004$, as accuracy to 3 decimal places only is required.

$$= 12{\cdot}012 - 4(1 + 0{\cdot}0005)^{-1}$$

$$= 12{\cdot}012 - 4(1 - 1 \times 0{\cdot}0005 + \ldots.),$$

expanding the last term by the Binomial Series.

As 0·0005 is very small compared with unity, the next term in the expansion may safely be neglected.

$$\text{Thus,} \quad y \simeq 12{\cdot}012 - 4 \times 0{\cdot}9995,$$

$$= 12{\cdot}012 - 3{\cdot}998,$$

$$= 8{\cdot}014.$$

When $x = 2{\cdot}00001$,

$$y = 3 \times 2{\cdot}00001^2 - \frac{8}{2{\cdot}00001},$$

$$\simeq 3 \times 4{\cdot}00004 - 4(1 + 0{\cdot}000005)^{-1}.$$

Again, as accuracy to 5 decimal places only is required, it is safe to approximate the square of 2·00001 as shown.

$$\therefore \quad y = 12{\cdot}00012 - 4(1 - 1 \times 0{\cdot}000005 + \ldots.),$$

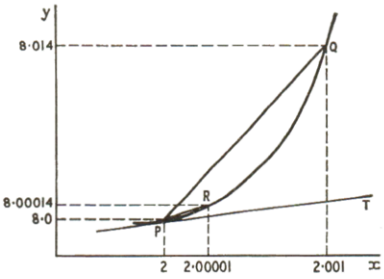
expanding the last term by the Binomial Series.

In a similar way to the first case when $x = 2{\cdot}001$, it is safe to neglect all terms except the first for 5 decimal-place accuracy.

$$\therefore \quad y \simeq 12{\cdot}00012 - 4 \times 0{\cdot}999995,$$

$$= 12{\cdot}00012 - 3{\cdot}99998,$$

$$= 8{\cdot}00014.$$



(*a*)

The points P, Q and R are illustrated in the sketch (*a*) in which the relative magnitudes are exaggerated for the sake of clarity.

$$\text{The slope of the line PQ} = \frac{8{\cdot}014 - 8}{2{\cdot}001 - 2}$$

$$= \frac{0{\cdot}014}{0{\cdot}001} = \underline{14}$$

$$\text{The slope of the line PR} = \frac{8{\cdot}00014 - 8}{2{\cdot}00001 - 2}$$

$$= \frac{0{\cdot}00014}{0{\cdot}00001} = \underline{14}$$

The slopes of PQ and PR are correct to at least 3 and 5 decimal places, respectively. The inference is that the slope of the tangent to the curve at point P is exactly 14.

(*b*)

$$y = 3x^2 - \frac{8}{x}.$$

Suppose x increases by a small amount δx and let δy be the corresponding change in y.

$$\text{Then,} \quad y + \delta y = 3(x + \delta x)^2 - \frac{8}{x + \delta x},$$

$$\text{or,} \quad \delta y = 3(x + \delta x)^2 - \frac{8}{x + \delta x} - \left(3x^2 - \frac{8}{x}\right),$$

$$\text{since,} \quad y = 3x^2 - \frac{8}{x},$$

$$\therefore \quad \delta y = 3x^2 + 6x\delta x + 3\delta x^2 - 3x^2 + \frac{8}{x} - \frac{8}{x+\delta x},$$

$$= 6x\delta x + 3\delta x^2 + \frac{8(x+\delta x) - 8x}{x(x+\delta x)},$$

$$= 6x\delta x + 3\delta x^2 + \frac{8\delta x}{x(x+\delta x)}.$$

$$\therefore \quad \frac{\delta y}{\delta x} = 6x + 3\delta x + \frac{8}{x(x+\delta x)}.$$

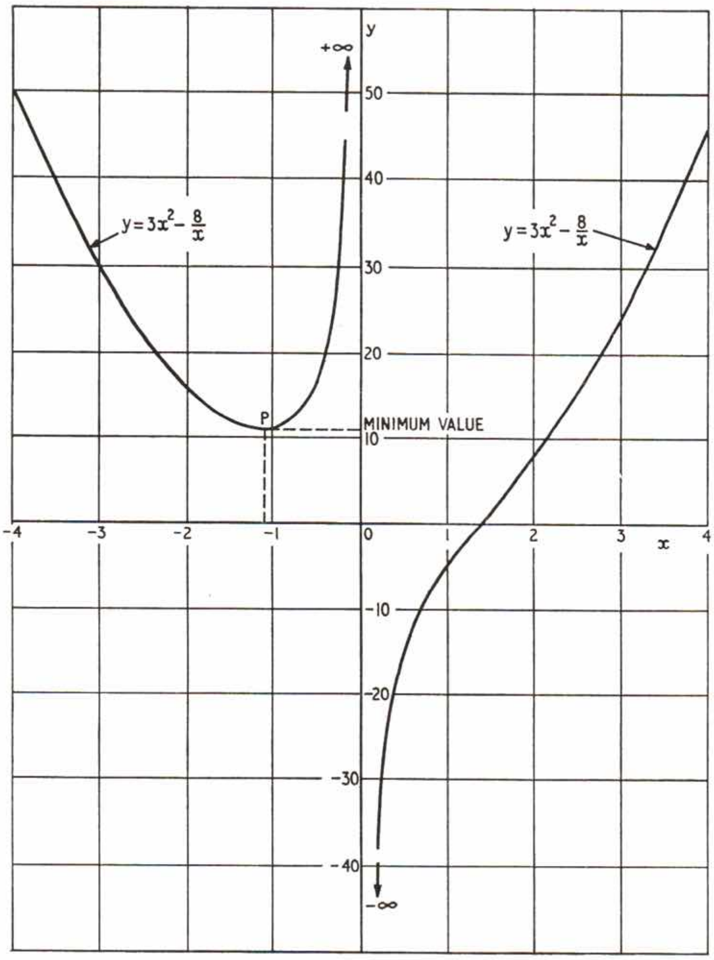
$$\text{Now,} \quad \frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

$$= \underline{6x + \frac{8}{x^2}}$$

Note.—When $x = 2$, (point P),

$$\frac{dy}{dx} = \text{gradient of tangent to the curve at P}$$

$$= 12 + \frac{8}{4} = 14 \text{ as inferred from part } (a)$$



(*b*)

The curve is shown in sketch (*b*). It comprises two parts as shown, each part being asymptotic to the y-axis as x tends to zero. There is a minimum value of approximately 10·9 at point P when $x \simeq -1{\cdot}1$ and a point of inflexion at $y = 0$, when $x \simeq 1{\cdot}39$.

***Q.* 9.** (*a*) *Evaluate* (*i*) $\int_1^4 \frac{x-1}{\sqrt{x}}\,dx$, (*ii*) $\int_0^{\frac{\pi}{3}} \sin x\,dx$.

(*b*) *Calculate the mean value of the function* $\frac{x^2}{4} - \frac{4}{x^2}$ *from* $x = 2$ *to* $x = 4$.

A. 9. (*a*) (*i*) $\int_1^4 \frac{x-1}{\sqrt{x}}\,dx$,

$$= \int_1^4 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx,$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right]_1^4,$$

$$= 2\left[\frac{4^{\frac{3}{2}}}{3} - 4^{\frac{1}{2}} - \frac{1}{3} + 1\right],$$

$$= 2\left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right),$$

$$= 2\left(\frac{7}{3} - 1\right) = 2 \times \frac{4}{3} = \underline{\frac{8}{3}}.$$

(*ii*) $\int_0^{\frac{\pi}{3}} \sin x\,dx = \left[-\cos x\right]_0^{\frac{\pi}{3}}$,

$$= -\cos\frac{\pi}{3} + \cos 0,$$

$$= -\frac{1}{2} + 1 = \underline{\frac{1}{2}}.$$

(*b*) Mean value of $\frac{x^2}{4} - \frac{4}{x^2}$,

$$= \frac{\int_2^4 \left(\frac{x^2}{4} - \frac{4}{x^2}\right) dx}{4-2},$$

$$= \frac{\int_2^4 \left(\frac{x^2}{4} - 4x^{-2}\right) dx}{2},$$

$$= \frac{1}{2}\left[\frac{x^3}{12} - \frac{4x^{-1}}{-1}\right]_2^4,$$

$$= \frac{1}{2}\left[\frac{x^3}{12} + \frac{4}{x}\right]_2^4,$$

$$= \frac{1}{2}\left(\frac{64}{12} + 1 - \frac{8}{12} - \frac{4}{2}\right),$$

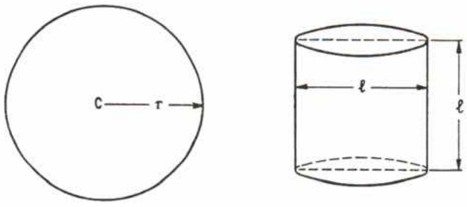
$$= \frac{1}{2}\left(\frac{16}{3} + 1 - \frac{2}{3} - 2\right),$$

$$= \frac{1}{2}\left(\frac{14}{3} - 1\right) = \frac{1}{2} \times \frac{11}{3} = \underline{\frac{11}{6}}.$$

***Q.* 10.** (*a*) *A solid sphere is re-cast without loss of material to form a solid cylinder whose diameter equals its axial length. Calculate as a percentage the resulting increase in surface area.*

(*b*) *Sketch the graph of the curve whose equation in polar coordinates is* $r^2 = a^2 \cos 2\theta$, *for values of* θ *from* 0 *to* π *radians. For what range(s) of values of* θ *is there no curve?*

A. 10. (*a*) Let r be the radius of the sphere and let l be the length and diameter of the cylinder. See sketch (*a*).



(*a*)

MATHEMATICS B, 1966 *(continued)*

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$= \text{volume of cylinder},$$

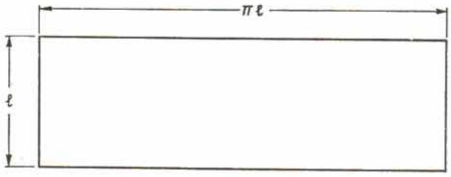
$$= \pi\left(\frac{l}{2}\right)^2 l\,.$$

$$\therefore\quad \frac{4}{3}\pi r^3 = \frac{\pi l^3}{4},$$

$$\text{or}\quad l^3 = \frac{16r^3}{3}\,.$$

Surface area of sphere $= 4\pi r^2$.

Surface area of cylinder $= 2\pi\left(\frac{l}{2}\right)^2 + \pi l^2$, since the curved surface of the cylinder = circumference of circle of diameter l × axial length l (see sketch (*b*)).



(*b*)

$$\therefore\quad \text{Surface area of cylinder} = \frac{\pi}{2}l^2 + \pi l^2 = \frac{3\pi l^2}{2}.$$

$$\text{But}\quad l = r\sqrt[3]{\frac{16}{3}} = r\,\frac{16^{\frac{1}{3}}}{3^{\frac{1}{3}}}\,.$$

$$\therefore\quad l^2 = r^2\,\frac{16^{\frac{2}{3}}}{3^{\frac{2}{3}}}.$$

$\therefore$ Surface area of cylinder,

$$= \frac{3\pi}{2}\,r^2\,\frac{16^{\frac{2}{3}}}{3^{\frac{2}{3}}},$$

$$= \pi r^2\,\frac{3^{\frac{1}{3}}}{2^{\frac{1}{3}}}\,8^{\frac{2}{3}},$$

$$= \pi r^2\,\frac{1{\cdot}442}{1{\cdot}260}\,4,$$

$$= 4\pi r^2\,1{\cdot}145\,.$$

No.	*Log.*
1·442	0·1590
1·260	0·1004
	0·0586

Hence, increase in surface area over that of sphere

$$= \frac{1{\cdot}145 - 1}{1} \times 100 \text{ per cent},$$

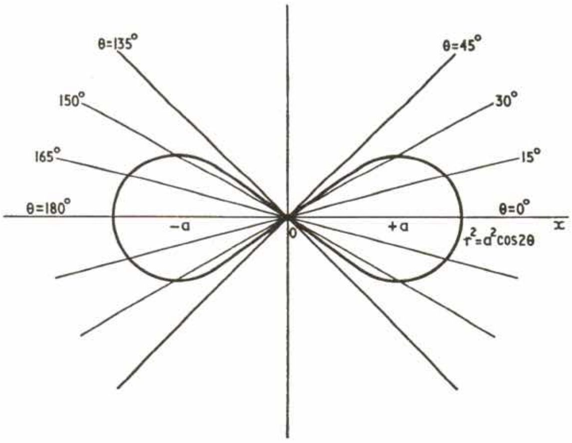
$$= \underline{14{\cdot}5 \text{ per cent.}}$$

(*b*)

$$r^2 = a^2\cos 2\theta,$$

$$\text{or,}\quad r = \pm\,a\sqrt{\cos 2\theta}.$$

The graph may be sketched from the following table of values. See sketch (*c*).



(*c*)

$\theta°$	0	15	30	45	135	150	165	180
$2\,\theta°$	0	30	60	90	270	300	330	360
$\cos 2\theta$	1	0·866	0·5	0	0	0·5	0·866	1
$\sqrt{\cos 2\theta}$	±1	±0·931	±0·707	0	0	±0·707	±0·931	±1
r	±a	±0·931a	±0·707a	0	0	±0·707a	±0·931a	±a

As θ varies from 0 to π radians, i.e. from 0° to 180°, 2θ varies from 0° to 360°. Between 90° and 270°, however, cos 2θ is negative and hence $\sqrt{\cos 2\theta}$ will be the square root of a negative number, i.e. it is imaginary and hence there is no curve for the range of θ from 45° to 135° or from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$ radians.

TELECOMMUNICATION PRINCIPLES B, 1966

Students were expected to attempt three questions from *Q*. 1–4 and three from *Q*. 5–10.

***Q*. 1.** *Write down the frequency, the r.m.s. and the peak values of a voltage wave expressed as:*

$$v = 14{\cdot}1 \sin 1{,}000\pi t$$

Write down expressions for the current flowing when this voltage is applied across

(*a*) *a 5-ohm resistor,*
(*b*) *a 1-mH inductor of negligible resistance,*
(*c*) *a 150-μF capacitor.*

Sketch the waveforms of these currents, showing clearly:

(*a*) *the phase relationship of each current to the applied voltage,*
(*b*) *the peak value of each current.*

A. 1. The general expression for a sinusoidal voltage v of frequency f c/s, with peak value V at an instant of time t seconds, is given by,

$$v = V\sin 2\pi f t.$$

Thus, for the expression in the question,

$$\text{frequency, } f = \underline{500 \text{ c/s}}$$

$$\text{peak voltage, } V = \underline{14{\cdot}1 \text{ volts,}}$$

$$\text{r.m.s. voltage} = \frac{V}{\sqrt{2}} = \underline{10 \text{ volts.}}$$

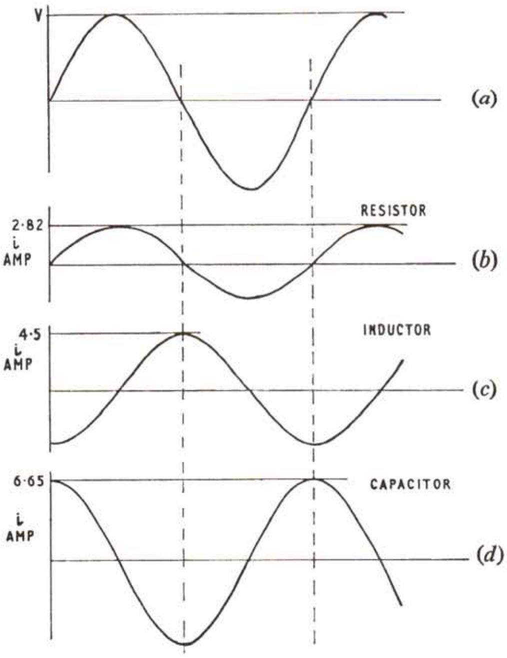
The equivalent expression for the current is, $i = I\sin 2\pi f t$ which becomes $I\sin\left[2\pi f t + \frac{\pi}{2}\right]$ if the current leads the voltage and $I\sin\left[2\pi f t - \frac{\pi}{2}\right]$ if the current lags the voltage.

(*a*) When the voltage v is across a 5-ohm resistor, the current will be $i = \frac{v}{5} = \frac{14{\cdot}1}{5}\sin 1{,}000\pi t$

$$\text{i.e. } \underline{i = 2{\cdot}82 \sin 1{,}000\pi t}$$

The current will be in phase with the voltage v (see sketch (*b*)).

(*b*) If the resistor is replaced by an inductor of 1 mH, then the current will lag the voltage by $\frac{\pi}{2}$ radians (see sketch (*c*)).



The reactance of 1 mH at 500 c/s $= 1{,}000\pi \times 10^{-3}$
$= +\ \pi$ ohms reactive.

$$\therefore i = \frac{14{\cdot}1}{\pi} \sin\left[1{,}000\pi t - \frac{\pi}{2}\right],$$

$$\text{and} \quad \underline{i = 4{\cdot}5 \sin\left(1{,}000\pi t - \frac{\pi}{2}\right).}$$

(*c*) If the inductor is replaced by a 150 μF capacitor, the current will lead the voltage across it by $\frac{\pi}{2}$ radians (see sketch (*d*)).

The reactance of 150 μF at 500 c/s $= \dfrac{10^6}{150 \times 1{,}000\pi} = \dfrac{100}{15\pi}$ ohms

$$\therefore i = \frac{14{\cdot}1}{2{\cdot}12} \sin\left[1{,}000\pi t + \frac{\pi}{2}\right],$$

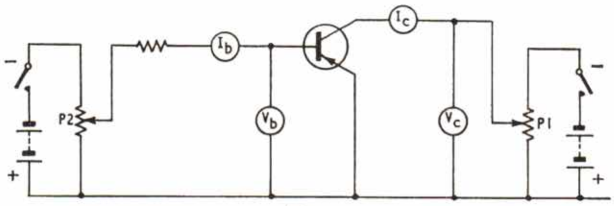
$$\underline{= 6{\cdot}65 \sin\left[1{,}000\pi t + \frac{\pi}{2}\right].}$$

***Q.* 2.** *Sketch a test circuit for obtaining the collector-current–collector-voltage characteristics of a transistor. Give a brief description, with typical resulting curves, of an experiment determining these characteristics for a transistor connected in the following configurations:*

(a) common emitter and (b) common base.

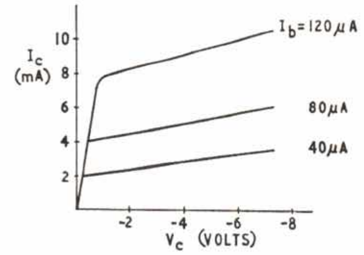
Sketch simple transistor amplifier circuit for either (a) or (b), showing the polarities of the battery connexions.

A. 2. *Common-emitter connexion:* The test circuit for a transistor connected in common-emitter configuration is shown in sketch (*a*). For experimental work it is convenient to provide separate batteries for the input and output circuits; in both cases potential adjustment is provided by the sliding potentiometers P1 and P2.



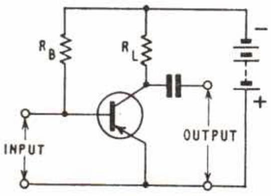
(*a*)

The collector-current/collector-voltage characteristics are plotted by obtaining corresponding readings for I_c and V_c, I_b being held constant for each set of readings. The procedure is as follows, P1 is adjusted to a suitable value of V_c and I_b is set to 40μA by using P2. The values of V_c and I_c are then recorded. P1 is then adjusted to give a higher value of V_c and P2 to keep I_b constant at 40μA. This procedure is repeated until enough readings have been taken to give a satisfactory curve as shown in sketch (*b*). The experiment is then repeated for larger values of I_b.



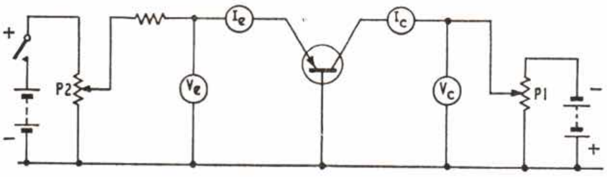
(*b*)

A simple form of amplifier circuit is shown in sketch (*c*). The bias potential between base and emitter is obtained by controlling the emitter–base–current with the resistor RB. The value of the resistance of the input circuit will also affect the bias potential.

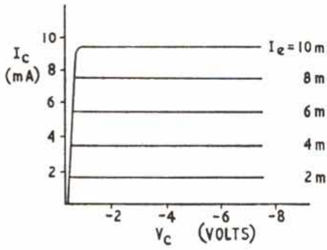


(*c*)

Common-base connexion: The test circuit for a transistor in common-base configuration is shown in sketch (*d*). The battery potential at the input is now negative with respect to the base, the reverse of that at the output. The procedure is as for the common-emitter circuit but I_e is now held constant for each set of V_c/I_c readings. The characteristics are nearly flat and parallel as shown in sketch (*e*).



(*d*)



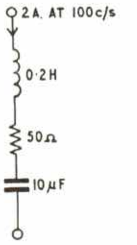
(*e*)

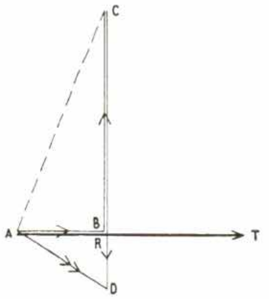
***Q.* 3.** *Use vectors drawn to scale to find the voltage across a circuit consisting of a coil of self-inductance 0·2 H and resistance 50 ohms connected in series with a capacitor of 10 μF, the circuit carrying a current of 2 amps at 100 c/s.*

Determine the voltage across the terminals of the inductor.

What is the phase angle between the current flowing and the voltage across the whole circuit?

A. 3. The current will be the same in each component as it is a series circuit. The current vector thus provides a convenient datum for the vector diagram.





The voltage across the 50-ohm resistor is in-phase with the current AT and is represented by AB, value 100 volt.

The current in the inductor lags the voltage by $\frac{\pi}{2}$ and is proportional to its reactance. It is represented in the diagram by BC.

Reactance of inductor $= 200\pi \times 0{\cdot}2 = 125{\cdot}6$ ohms.

Voltage, BC, $= 125{\cdot}6 \times 2 = 251{\cdot}2$ volts.

Similarly the voltage across the capacitor is proportional to its reactance, but this time the current leads the voltage by $\frac{\pi}{2}$ and is represented by vector CD on the diagram.

Reactance of capacitor $= \dfrac{10^6}{200\pi \times 10} = 159$ ohms

Voltage, CD, $= 159 \times 2 = 318$ volts.

The resultant voltage across the series circuit is represented by the vector AD.

In triangle ABD, AB $= 100$, BD $= 318 - 251{\cdot}2 = 66{\cdot}8$.
Then by measurement or calculation $\underline{\text{AD} = 120{\cdot}4 \text{ volts.}}$

The phase angle of the whole circuit $=$ BAD $= \tan^{-1}\dfrac{66{\cdot}8}{100} = \underline{33^\circ\, 45'}$

The voltage across the inductor is given by AC $= \underline{270 \text{ volt.}}$

Q. 4. *An uncharged capacitor (C) in series with a resistor (R) is connected to a constant voltage supply (V). Why does the voltage across the capacitor rise slowly?*

Write down an expression for the voltage across the capacitor after time (t) seconds. Sketch a curve of voltage against time.

What is the meaning of time constant for the circuit?

A capacitor of 10 μF is charged through a series resistor from a 100-volt battery. What value of resistor will give a time constant of 0·1 seconds?

For this circuit calculate the current flowing:

(a) at the instant of switching on,
(b) after a time equal to the time constant,
(c) after a long period of time.

A. 4. A voltage, V, can only exist between the plates of the capacitor C by virtue of the fact that the capacitor carries a charge Q. The relationship $V = Q/C$ then applies. The potential difference across a given charged capacitance is proportional to the charge carried. When an uncharged capacitor in series with a resistor is connected across a source of voltage the charging current is limited by the resistor in accordance with Ohm's law. The voltage across the resistor is the difference between the battery voltage and the p.d. across the capacitor. Time is required for the charge to enter the capacitor and i, the instantaneous current, is related to the charge passed, q, by the following expression, $q = \int_0^t i\, dt$. As more charge is acquired by the capacitor, its p.d. rises and the voltage across the resistor, R, falls. Hence the charging current falls as time increases. The law is,

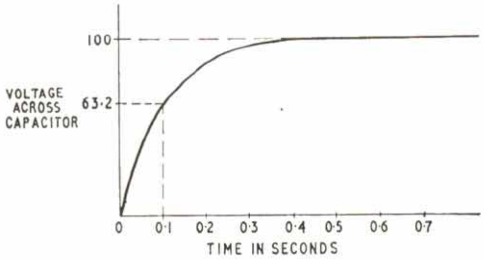
$$\underline{v = V\left(1 - e^{\frac{-t}{CR}}\right)},$$

where V is the battery potential and e is the base of natural logarithms.

The time constant is CR. This is defined as the time in seconds needed for the p.d. across the capacitor to rise from zero to 0·632 of its final value

When $t = CR$, then $v = V\left(1 - e^{\frac{-CR}{CR}}\right) = V\left(1 - e^{-1}\right) = 0{\cdot}632\, V.$

Note: This relation is useful because it provides an easy way of plotting the curve of V against t. Every additional intercept equal to CR on the time axis will give a vertical ordinate increasing in height by 63·2 per cent of the previous ordinate.



If $C = 10\ \mu$F and the time constant $= 0{\cdot}1$,
then, $CR = 10 \times 10^{-6} \times R = 0{\cdot}1$ seconds.
$\therefore\ R = 10^4 = \underline{10 \text{ k ohms.}}$

(a) At the instant of switching on, the full battery voltage will be across the resistor R because there is no p.d. across the capacitor.

$\therefore$ The initial current, $\underline{i_0, = 10 \text{ mA.}}$

(b) After 0·1 seconds the voltage across the capacitor $= 0{\cdot}632 \times V$
$= 63{\cdot}2$ volt.

$\therefore$ voltage across $R = 36{\cdot}8$ volt.

$\therefore\ i_{t.c.} = \dfrac{36.8}{10{,}000} = 3{\cdot}7$ mA.

(c) After an infinite time, there will be no p.d. across the resistor.

$\therefore\ \underline{i = 0.}$

Q. 5. *What is meant by the self-inductance of a coil. State the factors on which this depends.*

A coil dissipates 150 W when a battery maintains 30 volts across its terminals. The coil dissipates the same power when an a.c. supply of 60 volts at 50 c/s replaces the battery. Although the power dissipation is the same in each case why are the a.c. and d.c. voltages different?

Neglecting any core losses, calculate:

(a) the resistance of the coil,
(b) the current taken in each case,
(c) the phase angle between the voltage and current in the a.c. case,
(d) the inductance.

A. 5. The self-inductance of a coil is the property that causes an e.m.f. to be induced in it when the flux due to a current in the coil is changed. Self inductance is commonly written simply as inductance.

Inductance is proportional to the square of the number of turns and to the cross-sectional area of the coil.

As core losses are negligible in the given coil, the ohmic resistance of the coil is the only source of loss. This is the same whether a.c. or d.c. is flowing and the loss in each case is given by i^2R watts where R is the ohmic resistance and i amps the current. Hence the d.c. current supplied by the battery is equal to the r.m.s. value of the alternating current at 50 c/s.

The a.c. voltage exceeds the d.c. voltage because the coil possesses reactance at 50 c/s and the opposition to a.c. is given by the impedance, the vector combination of the resistance and reactance of the coil.

(a) For the d.c. case, power dissipated $= \dfrac{V^2}{R}$, where R is the resistance of the coil, or $150 = \dfrac{(30)^2}{R}$,

$\therefore\ \underline{R = 6 \text{ ohms.}}$

(b) For the d.c. case,

Power dissipated $= i^2 R.$

$\therefore\ i^2 = 25,$

and $i = \underline{5 \text{ amp.}}$

If the inductor is considered to be equivalent to a pure inductance in series with a resistance, R, the current through the combination is given by the current through the resistance. The value of this will thus be the same as the current in the d.c. case.

(*c*) The reactance of the coil $= 2\pi fL$ where f is the frequency (50 c/s) and L is the inductance of the coil.

The impedance of the coil is given by the expression $\sqrt{(2\pi fL)^2 + R^2}$. Under a.c. conditions the current flowing is 5 amps and the voltage across the circuit is 60 volts. Then,

$$60 = 5\sqrt{(100\pi L)^2 + 6^2}$$

$$L = \sqrt{108}/100\pi = 0{\cdot}0331 \text{ H.}$$

$$\text{Phase angle} = \tan^{-1}\frac{\omega L}{R}$$

$$= \tan^{-1}\left(\frac{100\pi \times 0{\cdot}0331}{6}\right) = 1{\cdot}73.$$

$$\therefore \text{Phase angle} = \underline{59^\circ\ 58'}$$

(*d*) From (*c*) above, inductance, L, $= \underline{0{\cdot}0331 \text{ H.}}$

Q. 6. *With reference to a thermionic triode valve, define the terms:*

(*a*) *anode slope resistance* (R_a),
(*b*) *amplification factor* (μ),
(*c*) *mutual conductance* (g_m).

A tuned-anode triode valve amplifier operates at resonance. For the triode, $\mu = 30$, $R_a = 15$ *K ohms; the anode load consists of a* 500-μH *inductor* (L) *of* 30-*ohm resistance* (R) *with a parallel-connected tuning capacitor* (C) *of* 450 *pF.*

Sketch the simple equivalent circuit of the amplifier and calculate its stage gain at resonance, given that the dynamic resistance of the resonant circuit is $\dfrac{L}{CR}$.

A. 6. (*a*) The anode slope resistance (R_a) of a triode valve is defined as the slope of the anode current–anode voltage characteristic measured with the grid voltage constant.

$$R_a = \frac{\delta V_a}{\delta I_g}$$

The anode slope resistance is some times called the anode a.c. resistance.

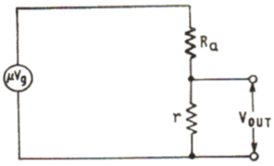
(*b*) The amplification factor, $\mu = \dfrac{\delta V_a}{\delta V_g}$, taken with I_a constant.

(*c*) The mutual conductance gm $= \delta I_a/\delta V_g$, with V_a constant. This is the slope of the I_a/V_g characteristic plotted with V_a constant.

The resonant circuit in the anode circuit of the simple triode amplifier behaves at resonance as a resistance of value $\dfrac{L}{CR}$ ohms,

$$\frac{L}{CR} = \frac{500 \times 10^{-6}}{450 \times 10^{-12} \times 30} = 37{,}000 \text{ ohms.}$$

The valve thus behaves as a generator in series with the anode-slope resistance and the anode load (see the sketch).



The input voltage V_g becomes μV_g in the equivalent circuit to represent the gain of the valve.

The current, i, in the equivalent circuit $= \dfrac{\mu V_g}{R_a + r}$, where r is the anode load.

The output voltage = the voltage developed across the anode load,

$$= ir = \frac{\mu V_g r}{R_a + r}.$$

$$\text{The gain of the stage} = \frac{\text{output voltage}}{\text{input voltage}},$$

$$= \frac{1}{V_g} \times \frac{\mu V_g r}{R_a + r},$$

$$= \frac{\mu r}{R_a + r},$$

$$= \frac{30 \times 37{,}000}{15{,}000 + 37{,}000} = \underline{21{\cdot}4.}$$

Q. 7. *Describe with the aid of sketches the principle of the simple d.c. generator. Explain*

(*a*) *the function of the commutator,*
(*b*) *the meaning of the term shunt-connected field coils.*

A 2-pole d.c. generator has a flux per pole in the air-gap of 5 *mWb. The rotating armature coil has* 240 *turns (i.e.* 480 *conductors) connected in series.*

Calculate the average e.m.f. at the brushes, when the machine is running at 1,000 *rev/min with negligible current load.*

A. 7. For a description of a simple d.c. generator see A.51, Model Answer Book, Telecommunications Principles B, 1965.

For the 2-pole d.c. generator in the problem:

Average e.m.f. = average rate of change of flux linkage,

= flux per pole × no. of poles × no. of turns × revs per second × reversals of flux per rev.

$$= 5 \times 10^{-3} \times 2 \times 240 \times \frac{1{,}000}{60} \times 2,$$

$$= \underline{80 \text{ volt.}}$$

The negligible load current implies that the voltage-drop within the armature coils due to copper losses can be neglected; the full 80 volts will thus appear across the brushes.

Q. 8. *Using the components shown in Fig.* 1, *describe a bridge suitable for measuring an unknown capacitance* C_x *at audio frequencies.*

What limitations must be set on the type of capacitor that can be measured with this circuit?

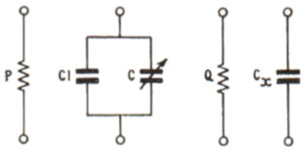


Fig. 1

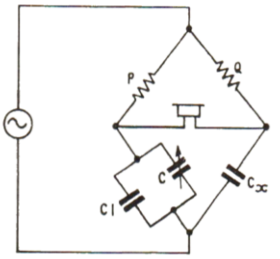
What type of detector would be suitable?

Give the balance conditions for the bridge.

Calculate the value of C_x *when* $P = Q = 100$ ohms, $C_1 = 0{\cdot}10\ \mu F$ *and a balance is obtained when* $C = 0{\cdot}025\ \mu F$.

How should the circuit of this bridge be altered to enable it to measure low grade capacitors equivalent to a capacitance in series with a resistance?

A. 8. The bridge circuit for measuring the capacitance C_x will be that shown in the sketch.



The two resistances are known as the ratio arms. For balance, no voltage must appear across the detector. This component could be a sensitive high impedance pair of earphones, or an amplifier with a meter or earphone to indicate the output voltage.

The conditions for balance are that the voltages across P and Q shall be equal and that the voltages across ($C_1 + C$) and C_x shall be equal and in-phase. Since no resistance is available to correct the phase relationship across ($C_1 + C$), only high grade capacitors with negligible loss can be measured.

At balance, no current flows in the detector.

If i_1 is the r.m.s. current in P and $(C_1 + C)$ and i_2 is the r.m.s. current in Q and C_x then,

$$i_1 P = i_2\ Q,$$

$$i_1 \times \frac{1}{\omega(C + C_1)} = i_2 \times \frac{1}{\omega C_1},$$

$$\text{or } C_x = \frac{P}{Q}(C + C_1),$$

$$\text{In the problem, } C_x = \frac{1{,}000}{1{,}000} \times (0{\cdot}125),$$

$$= \underline{0{\cdot}125\ \mu F.}$$

A low grade or lossy capacitor can be taken as equivalent to a capacitance C_x in series with a resistance R_x, then the arm $(C + C_1)$ will have to be modified by including an adjustable resistor, R.

The conditions for balance are,

$$R_x = R\frac{Q}{P}$$

$$C_x = C\frac{P}{Q}.$$

Q. 9. *Describe the principle of operation of a meter suitable for measuring current of a few milliamperes at audio frequencies.*

State the factors:

(a) that limit the sensitivity of the instrument,

and (b) that reduce its accuracy as the frequency increases.

A. 9. A metal rectifier meter would be suitable for audio frequency measurement of small currents. See A.58, Telecommunication Principles B, Model Answer Book, 1965.

Q. 10. *Distinguish between the terms reactance and impedance. Why is the impedance of an inductor always greater than its reactance, whereas for a good quality capacitor the reactance and impedance are almost equal?*

Sketch on one pair of axes the reactance-frequency curves for

(a) a 0·05-H inductor and

(b) a 2-μF capacitor.

Sketch a curve showing how the combined reactance varies with the frequency when the inductor and capacitor are connected in series. Hence explain the condition for resonance.

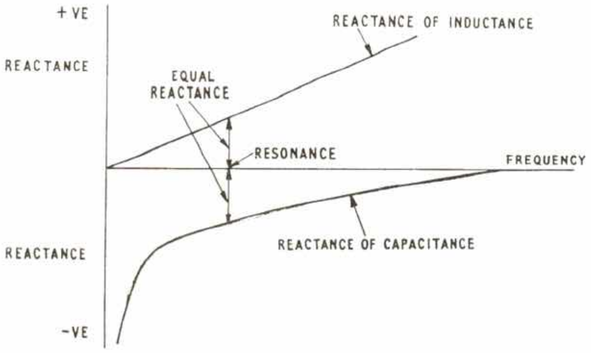
Calculate the frequency at which resonance occurs.

A. 10. In a circuit consisting of resistance, capacitance and inductance the vector representing an applied sinusoidal voltage will be displaced in phase from the vector representing the current flowing. The voltage vector can be resolved into two components mutually at right angles, one of which is in-phase with the current flowing and the other at right angles to the current, known as the quadrature component. The reactance, X, of the circuit is defined as the ratio of this quadrature voltage-component to the value of the current vector. The impedance, Z, of the circuit is the ratio of the total voltage vector to the current vector.

An inductor consists of a coil of wire and must therefore always possess some residual resistance. A high-grade capacitor consists, simply, of two plates enclosing a dielectric and has no conducting path to contribute a resistive component.

The reactance of an inductor of L henries $= 2\pi fL$ ohms.

Hence, the reactance X_L of an inductor is proportional to frequency, and the curve, shown in sketch (*a*) is a straight line through the origin with gradient denoting the inductance.

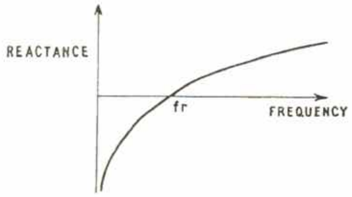


(*a*)

$$\text{The reactance of a capacitor of } C, \text{ farad } = \frac{1}{2\pi fC}.$$

The curve of X_C against frequency is shown in sketch (*a*). It consists of a rectangular hyperbola in the fourth quadrant.

The resonance condition is that the negative reactance due to the capacitance equals the positive reactance due to the inductance. The combined reactance is shown in sketch (*b*); the intercepts for the L and C reactances are equal when the combined reactance is zero,



(*b*)

$$\text{i.e}\quad 2\pi f_r L = \frac{1}{2\pi f_r C},$$

$$\text{Then } f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{10^4}{2\pi\sqrt{10}} = \underline{504 \text{ c/s}}$$

LINE PLANT PRACTICE A, 1966

Q. 1. *Describe in detail the process of providing a drop-wire cable from a distribution pole to a subscriber's premises across a road and above low-voltage power wires. Describe particularly the safety precautions which must be observed in carrying out the work.*

A. 1. A drop-wire cable may be erected above the low-voltage power wires, whilst the latter are live, by the "sash line without loops" method as described below.

The appropriate attachments for securing the drop-wire cable, i.e. brackets, hooks and clamps, should first be fitted to the subscriber's premises and to the pole. With a drum of drop-wire cable appropriately positioned on the ground, the free end of the drop-wire cable is fed over the hook or bracket at the subscriber's premises and returned to the ground. To obviate the possibility of any metallic contact occurring between the live power wires and the drop-wire cables in the subsequent operations, about 1 in. of the conductors of the latter should be removed from the free end without damaging the outer cover. A thin dry sash-line, of length greater than the distance between the subscriber's premises and the distribution pole, is then thrown over the power lines and the free end of the drop-wire cable is fastened to one end of the sash line. The other end of the sash line is taken to the top of the pole and secured. From the pole top the sash line is pulled taut to clear the power wires and then, whilst the unwinding of the drop-wire cable drum is carefully controlled, the drop-wire cable is pulled over and above the power wires to the pole top, to which it is temporarily secured. During this process the sash line and drop-wire cable must be maintained taut so that the latter does not rub on the power wires as it is pulled across. At the subscriber's end the drop-wire cable may then be secured to the previously fitted bracket or hook and run as necessary into the subscriber's premises. Finally it is tensioned by hand and terminated at the pole.

During the course of the above process, inconvenience to the public must be kept to a minimum. In particular, operations which impede pedestrians and traffic must not be commenced until it is safe to do so and traffic flow must be carefully controlled by means of red and green flags.

The following safety precautions must also be observed.

(*a*) Safety belts are to be worn by all men working at the pole top.

(*b*) The pole must be tested for safety before being ascended.

(*c*) Wooden ladders are to be preferred and if metal ladders, mechanical aids or ladder vehicles are used, special care must be exercised.

(*d*) Rubber gloves are to be worn if it is considered that there is any risk of accidental contact with low-voltage power, and also when handling wet or damp ropes in contact with the power wires.

(*e*) Warning notices should be displayed.

Q. **2.** *Describe in detail tests which are used to detect decay in a wood pole erected in a street. How would you decide whether or not a decayed pole should be renewed?*

What precautions should be taken when replacing a decayed pole?

A. 2. The usual tests for detecting decay in wood poles are as follows:

(*a*) *Hammer Test*: This test is for detecting internal decay. The pole is tapped with a light hammer and the note obtained indicates the condition within the pole at the point struck; a sound pole will give a good "ring", but a hollow note indicates extensive internal decay, and a dull or dead note indicates slight internal or surface decay.

(*b*) *Prodding Test*: This test is for detecting external decay. The surface of the pole is prodded lightly with a sharp pointed tool. Decayed wood will offer little or no resistance to penetration and will not grip the point.

The hammer test is applied all round the pole, at one inch intervals, as near the ground as possible and at every few inches to other parts of the pole within reach of the ground. The ground is excavated to expose 12 inches of the pole below the ground line, the pole cleaned and the hammer and prodding tests applied to the exposed surface. If decay has not yet been detected, the hammer test is then applied at frequent intervals all round the pole from the point where the first hammer tests left off and up to the top of the pole.

The decision whether or not to renew a decayed pole would depend on the extent of the decay. A pole in which the decay reaches to a depth greater than 2 in. or has decay for more than about one third of the circumference to a depth greater than 1 in. should be considered dangerous and renewed.

Two stout ladders should be lashed to a dangerous pole before anyone attempts to climb it.

All the usual safety precautions associated with the replacement of poles should be employed and additional precautions are necessary.

Where the new pole can be erected before the decayed pole is recovered it is usual to set the new pole close to the one to be replaced. Precautions have to be taken to prevent the old pole from collapsing into the new pole hole when the ground is disturbed. Additional temporary stays may be fitted and steel bars forming a splint should be driven into the ground around the pole and lashed to it to prevent it from shearing off. Alternatively, a ground stay may be provided by roping the pole to a couple of crowbars driven into the ground on the side of the pole remote from the new pole-hole.

When the new pole has been erected, additional support to the decayed pole can be provided by lashing the two poles together in several places before the work to change the wires over begins. This is often necessary as the overhead wires on the old pole afford it some measure of support.

If the new pole is erected some distance from the decayed pole and it is unsafe to climb the old pole because of extensive decay, then new wires are run from adjacent poles to the new pole and the old wires cut off one at a time on either side to prevent undue twisting or strain of the old pole.

When the replacement pole is erected alongside a decayed pole it must be temporarily stayed at the top and at ground level so that it cannot heel over into the old pole hole when the old pole is removed. The temporary stays are removed when the old hole has been firmly filled in.

Extensively decayed poles in danger of collapse may be lowered simply by pulling on the temporary stays or by sawing through the pole below the ground line; bolster poles or ladders should be removed after changing over.

In exceptional circumstances, as for example, in a confined space where it is impossible to provide temporary stays or a derrick, and lowering in sections is the only practical method of recovering an extensively decayed pole, tubular steel scaffolding may be employed to provide a suitable staging around the pole.

Q. **3.** *What factors influence the siting of a distribution pole?*

Describe in detail a modern method of overhead distribution to subscribers' premises.

A. 3. The following factors should be considered in siting a distribution pole (D.P.):

(*a*) A site should be chosen so that the maximum number of tenancies can be served radially from the pole without additional fixtures or poles. If additional poles are required to serve the D.P. area, care should be taken to site the D.P. so that if required any subsidiary poles can be converted to D.P.s in the future.

(*b*) The position should be such that all tenancies can be served without difficulties due to trees, buildings or power wires. Care should also be taken to avoid wires crossing close to windows.

(*c*) The pole should not obstruct entrances to property, or views from windows and should not create a hazard to pedestrians or vehicular traffic. It should be clear of lamp standards and road signs.

(*d*) It should be sited, if possible, on the public footway and in a position which will not be affected by future road-widening schemes. Its exact position will depend on the wishes of the local authority, since some authorities prefer the pole sited on the fence line, whilst others prefer the kerb line.

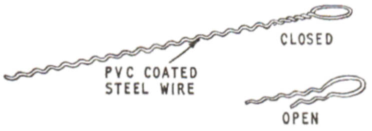
(*e*) Consideration should be given to other undertakers' plant and the ease of erection and maintenance of the pole and wires. A site near to an existing underground track would be an advantage.

A modern method of overhead distribution to subscribers' premises is by the use of insulated drop-wire from a ring-type distribution pole.

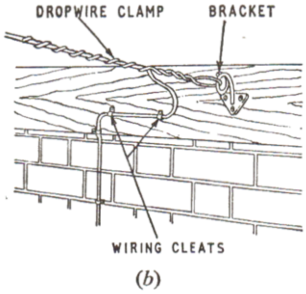
The latest type of drop-wire has steel conductors of 0·028 inches diameter with a copper coating. The conductors are laid up as a flat pair and insulated with grey PVC in a double 'D' formation. The drop-wire is run in one continuous length from a terminal block on the pole to a terminal block in the subscriber's premises.

The underground cable is cleated to the distribution pole and terminated on a terminal block fitted approximately 15 in. from the top of the pole. A split-ring-type pole-head, consisting of two half rings of $\frac{1}{2}$ in. galvanized mild steel is fixed to the pole approximately 6 in. above the terminal block. The pole-head ring is secured by means of an arm bolt, which carries spacing washers of suitable length to position it correctly.

The drop-wire is first attached to the subscriber's premises using a bracket and a drop-wire clamp. The bracket consists of a small flat plate to which is welded a spiral eye. The drop-wire clamp is an 18 in. stainless-steel wire helix coated with PVC to make it grip the cable, sketch (*a*). There is a plain loop at one end of the wire with a short helical tail, which is twisted back on to the main helix to form a closed eye for fixing to the house bracket. The eye of the clamp is first closed and the wire is then fixed by winding it round the helix of the clamp. Sufficient wire is left for feeding into the subscriber's premises before fixing it to the clamp. The wire is secured to the building by means of cable-fixing nails on masonry insulating, or staples on woodwork and is terminated in the subscriber's premises on a terminal block, see sketch (*b*).

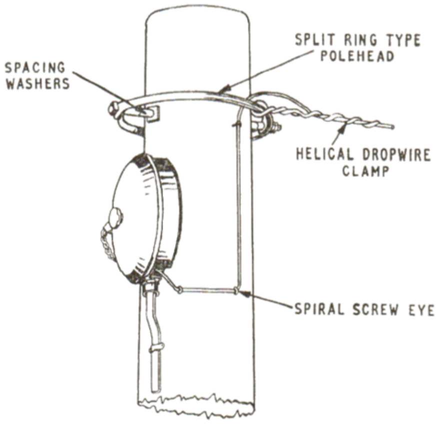


(*a*)



(*b*)

The other end of the wire is fed to the distribution pole, and after tensioning by hand, is fixed to the pole-head ring by means of a similar clamp to that used at the subscriber's premises. The wire is led over the pole-head ring, down the pole through spiral screw eyes and into the terminal block, where it is terminated on the appropriate underground-cable pair, as shown in sketch (*c*).

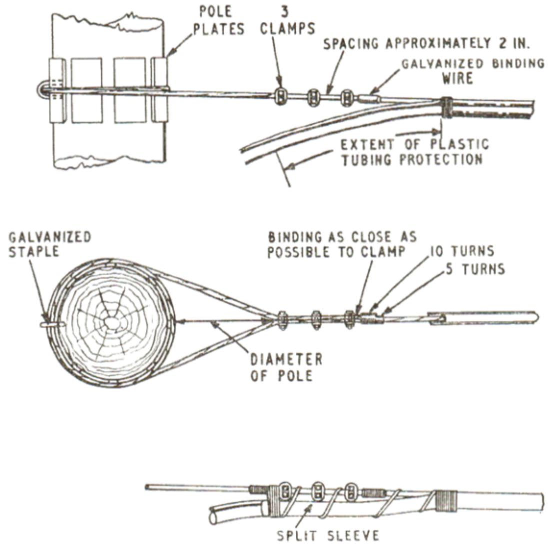


(*c*)

Q. 4. Describe with the aid of sketches three different methods of fixing a 15-pair 20-lb. per mile "figure-of-eight" type of self-supporting aerial cable to a pole. Explain the circumstances in which each of the methods is used.

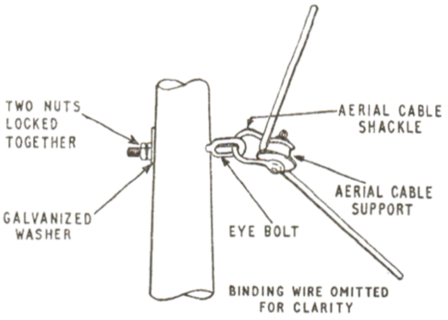
A. 4. The first method of fixing is described below. The suspension wire is first separated from the cable for a length equal to the length of the termination, plus the length down the pole, sufficient to enable a joint to be made. A sharp knife is used to slice off the top of the PVC covering from the wire and the suspension wire is pulled out of its sheathing, which must not be trimmed off. Several turns of binding wire are applied to the cable where the suspension wire leaves the sheath to prevent the cable from tearing off the suspension wire at this point.

Pole plates are fixed to the pole at the required height and the suspension wire taken twice round the pole, and secured by three clamps applied at 2 in. intervals with the first at a distance equal to the pole diameter away from the pole. A staple is then hammered into the pole to maintain the termination at the correct height. The spare suspension wire is cut off and the remaining end bound to the main strand. A length of split plastic tubing 2 in. in diameter is fitted over the cable as shown in sketch (*a*) and bound to the suspension wire to prevent the clamps from chafing the cable sheath. This method is used at the beginning and end of each section, at poles where the pull-on-pole is 30 feet or more and also at jointing points.



(*a*)

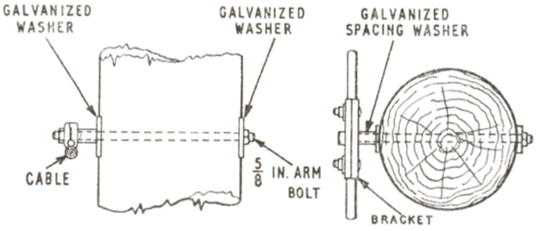
The second method of fixing is described below. The pole is bored for an eye bolt which is fitted with its eye in the horizontal plane. The shackle is assembled in the eye of the bolt with the cable support assembled on the shackle pin. The head of the pin and also the smaller flange of the cable support are uppermost as shown in sketch (*b*).



(*b*)

After the cable has been erected and tensioned it is firmly bound to the support with PVC insulated binding-wire. With the cable laying snugly in the support (suspension wire upwards) the middle of a 3 ft. length of binding wire is laid across the cable to one side of the support and three close turns made round the cable. A similar binding is made on the other side of the support. Each pair of wires is then pulled tight, taken round the underside of the cable support and made off on the cable, the PVC being stripped from the binding wire and the wires twisted together. This method is used at angle poles where the pull-on-pole is between 5 and 30 ft and at canal, building and road crossings.

The third method of fixing is used at straight-through positions. The cable is secured to the pole with a bracket which is assembled on a bolt with a spacing and two galvanized washers. The bracket has a groove along its length which accommodates the insulated suspension wire and clamps it tightly. This method of fixing is used at straight-through positions where the pull-on-pole is 5 ft. or less and is shown in sketch (*c*).



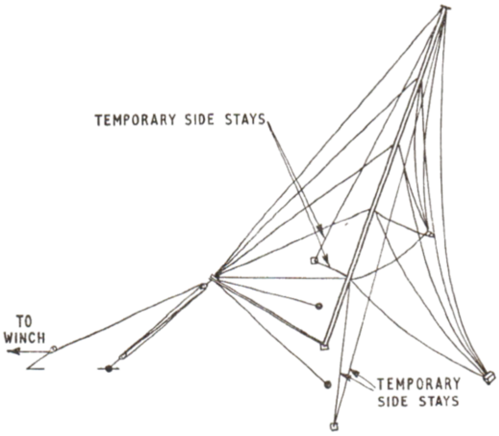
(*c*)

Q. 5. Describe the "falling derrick" method of erecting a light steel mast for a radio aerial.

A. 5. A concrete foundation block and accurately-positioned stay anchors are first provided. The mast is then assembled horizontally on supports, with its base universal joint attached to the base block and lying across the back-stay anchors. The stays and halyards are then attached, the side stays being made off, just taut, to their anchors and the back stays attached to their anchor at the approximate length. A tubular-steel derrick is then attached via a swivel joint to the base block or to the base of the mast and laid out horizontally at an angle slightly less than 90° to the mast. The remaining stays are attached tautly to the derrick head by means of clamps. A winch is then brought into position in the line of the mast and anchored. Its rope is passed through a block on an erection anchor or a stay anchor if suitable, and then attached to the top of the derrick. The derrick is then hoisted to the vertical and held by temporary side-stays. The general arrangement is shown in the sketch.

The winch then takes the load and the derrick will fall towards the ground as the mast rises. As the mast leaves the ground the winch must be frequently stopped and the mast checked for straightness and, if necessary, the supporting wires must be adjusted. On uneven ground the side stays must also be frequently adjusted.

As the mast nears the vertical the back stays can be adjusted, and then the remaining stays are transferred, one by one, from derrick to anchors. Finally the mast is checked for verticality and straightness, the stays are adjusted, the halyards checked for freedom, and the ends of the rigging screws are protected against corrosion by painting with a preservative compound.



Q. 6. *What precautions must be taken when mixing and placing concrete?*

Describe the process of excavation for, and the construction of, a medium size reinforced-concrete surface joint box.

A. 6. When mixing and placing concrete the following precautions must be taken:

(*a*) The quality of the ingredients should be carefully inspected. It is important that all the ingredients (cement, aggregate, sand, water) should be of high quality and special attention should be paid to cleanliness at all stages of the mixing and placing. Cement may be considered good if it feels warm, silky and free from lumps. Aggregate should be clean, screened river ballast, gravel, stone or other approved material of the nature of cubes (not flakes), and well graded in size. It should be free from dirt, floury stone dust, loam and earthy or like materials. Sand should be obtained from a river or pits and should be clean, sharp, gritty and free from loam, organic matter and adherent coatings. The water used must be clean and fit for human consumption.

(*b*) The ingredients should be measured in gauging boxes or by other means, and mixing may be done by machine or manually. The mixing must be thorough. When manual mixing is employed, the platform on which the operation is performed should be large enough to permit the material to be moved completely from one place to another in the course of being turned over.

Before adding the water, the dry material should be repeatedly turned over until the mixture is an even colour throughout. Mixing should then be continued while gradually adding the water, until the correct proportion of water is included. The whole mass should then be turned until it is thoroughly mixed.

The water should be applied through a rose or sprinkler only to the extent required to obtain a wet mixture with a plasticity that will permit convenient placing and compacting throughout the structure.

(*c*) Normal Portland-cement concreting should not be undertaken when the temperature is below 40°F. High-alumina cement concrete-work can be carried out successfully at temperatures down to 12°F. While mixing, the aggregate and sand is kept at a temperature above the freezing point of water and the mixing water used may, with advantage, be lukewarm. The concrete, when placed, is protected against frost for at least four hours; the protection may then be dispensed with, since, whilst setting, the concrete will generate sufficient heat to prevent the frost damaging the cement.

(*d*) In warm or windy weather, concrete should be prevented from drying too rapidly by shielding it from the sun and wind. Portland-cement concrete should be kept moist for 7 days (3 days in the case of rapid-hardening Portland cement and 24 hours for high-alumina cement) by frequent sprinkling with a hose or watering can, or by covering the concrete with a saturated sackcloth or wet sand.

(*e*) Concrete should be placed (not thrown) in position as quickly as possible after being mixed and should be lightly tamped and worked to fill all cavities. Concrete, when mixed, should be used within 30 minutes.

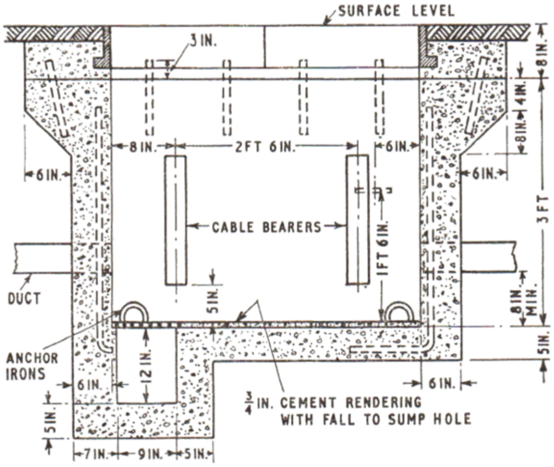
Building a reinforced-concrete surface joint box is carried out as follows:

After the position for the box has been determined, a pilot hole should be excavated to the limits of the finished joint-box in each of the four directions. Precautions to protect the workmen and public from danger should be taken. Having fixed the final position by examining the pilot hole, excavation of the ground to the full extent required may proceed. The full extent is 12 in. wider and longer than the finished inside dimensions of the joint-box and about 6 in. deeper than the finished floor level. Additional excavation will be required for the sump-hole. Excavation work may be done manually or with mechanical aids according to the situation and condition of the soil. In loose, very wet or unstable soil, wooden shuttering may be required to prevent the sides of the excavation from falling in; but in solid ground shuttering will not be necessary.

When the hole has been excavated to the required dimensions the floor should be rammed hard; hardcore may be necessary for this purpose. Laying of the concrete floor (including the sump-hole) may now commence, the concrete being placed, not thrown, into position and well tamped. Reinforcing bars for the floor and the floor-to-wall connexions, and for anchor irons, are laid in their appropriate positions and the floor is allowed to set.

The first part of the wall shuttering is now placed in position and neat cement mortar is laid on the base where the walls are to be built. Concreting of the walls and placing of the reinforcing bars, anchor rings, steps, bearer bolts, duct entries etc., now proceed stage by stage up to the top level of the joint-box. The concrete is well tamped and consolidated as the work proceeds. The frame for the cover is bedded into the top of the walls at the appropriate level of the roadway or footway.

The concrete is now allowed to set, taking the normal precaution (see first part of answer) and when set the shuttering is removed; the whole of the inside of the joint box is rendered with a mixture of 3 parts of sand and 1 of cement.



The sketch shows a typical joint-box.

Q. 7. *Describe the constructional differences between a 100-pair unit-twin paper-insulated cable suitable for subscribers' circuits and a star-quad cable of comparable size suitable for longer distance audio circuits. Give reasons for the differences.*

A. 7. A star-quad cable of comparable size to a 100-pair unit-twin cable would have 104 pairs.

The constructional differences between a 100-pair unit-twin paper insulated cable (P.C.U.T.) and a 104-pair star-quad cable (P.C.Q.T.) are as follows:

(*a*) *Insulation.* In the quad-type cable the insulation of the wires includes a helical lapping of string between the wire and the paper insulation. The lapping of string is not needed in the twin-type cable.

(*b*) *Quadding and twinning.* In the quad-type cable four insulated wires are formed into a group around a central string, twisted as a group and lapped with a coloured cotton. The wires of the pairs are diametrically opposite in the quad formation. In the unit-twin type cable two wires are twisted together to form a pair.

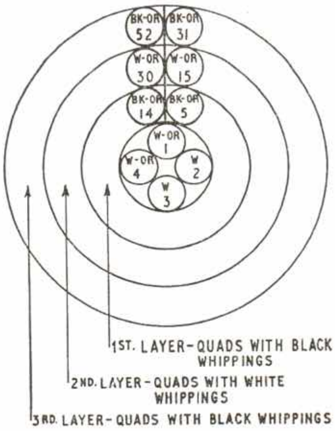
(*c*) *Stranding.* The quads of the quad-type cable are stranded into four layers consisting of four quads in the centre layer, each subsequent layer increasing by six quads with 22 quads in the outer layer. The direction of stranding is opposite for alternate layers. The outer layer is wrapped with at least two thicknesses of paper insulation.

In the unit-type cable the 100-pair cable is made up of two concentric units of 51 pairs, the additional pair in each unit is provided as a spare to replace faulty pairs if necessary. The pairs of the first unit are stranded into layers consisting of four pairs in the centre layer, 10 pairs in the first layer, 16 pairs in the second layer and 21 pairs in the third layer. The second unit comprises the 4th layer with 24 pairs and the 5th layer with 27 pairs. Adjacent layers are stranded in alternate directions and a whipping of at least three cottons or rayons is applied over each layer, except the outer. The outer layer is wrapped with insulating paper.

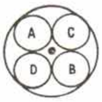
(*d*) *Wire identification.* In the quad-type cable, the wire is identified by markings on the paper which are coloured red or blue, the A, B, C and D wires being marked with one, two, three or four rings, respectively. The quads are identified by the colour of the whipping cotton and the colour of the paper markings in accordance with the following table:

Position of Quad in Layer	*Colour of Paper Markings*
1st (Marker)	Red
2nd, 4th etc.	Blue
3rd, 5th etc.	Red
Last (Reference)	Blue

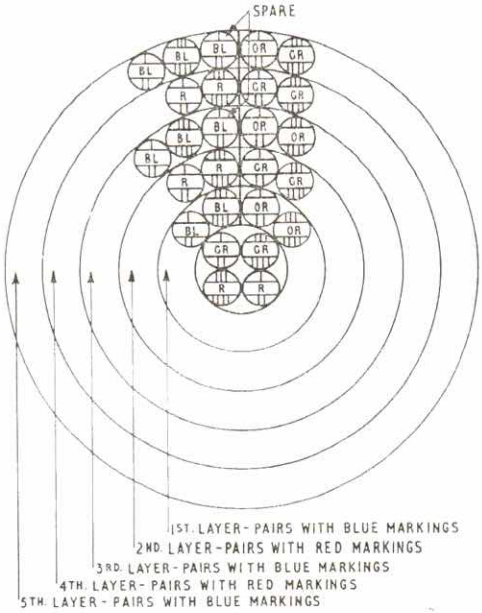
An abbreviated target diagram of the cable is shown in sketch (*a*) which indicates the colour of the quad whippings, white (W), orange (OR), or black (BK). The position of the conductors in the quad is shown in sketch (*b*). In the unit-type cable the paper markings are coloured red (R), blue (BL), green (GR), or orange (OR), the A and B wires of the odd pair and A and B wires of the even pair being marked with one, two, three or four rings, respectively. An abbreviated target diagram of the cable is shown in sketch (*c*).


BK-OR 52
BK-OR 31
W-OR 30
W-OR 15
BK-OR 14
BK-OR 5
W-OR 1
W-OR 4
W 2
W 3
1ST. LAYER-QUADS WITH BLACK WHIPPINGS
2ND. LAYER-QUADS WITH WHITE WHIPPINGS
3RD. LAYER-QUADS WITH BLACK WHIPPINGS

(*a*)



(*b*)


SPARE
BL
OR
GR
R
1ST. LAYER - PAIRS WITH BLUE MARKINGS
2ND. LAYER - PAIRS WITH RED MARKINGS
3RD. LAYER - PAIRS WITH BLUE MARKINGS
4TH. LAYER - PAIRS WITH RED MARKINGS
5TH. LAYER - PAIRS WITH BLUE MARKINGS

(*c*)

(*e*) *Size.* The quad-type cable is marginally smaller in diameter and sheath thickness than the unit-type cable.

The differences in construction are largely necessitated by crosstalk limitations which are naturally more severe for cables carrying long distance circuits. Crosstalk between pairs is largely governed by the capacitance unbalances between them and the latter must, therefore, be kept within close limits for cables to be used for long-distance circuits. Capacitance unbalances between wires and earth also result in interference from power circuits being introduced into the telephone system and for this added reason they must be kept within specified limits.

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